Chapter 7

The Normal Probability Distribution

Section 7.1

1. For the graph to be that of a probability density function, (1) the area under the graph over all possible values of the random variable must equal 1, and (2) the graph must be greater than or equal to 0 for all possible values of the random variable. That is, the graph of the equation must lie on or above the horizontal axis for all possible values of the random variable.

2. distribution; density

3. The area under the graph of a probability density function can be interpreted as either:
   (1) The proportion of the population with the characteristic described by the interval; or
   (2) The probability that a randomly selected individual from the population has the characteristic described by the interval.

4. We standardize the normal random variables so that one table can be used to find the area under the curve of any normal density function.

5. $\mu-\sigma$; $\mu+\sigma$

6. As σ increases, the height of the graph of the normal density function decreases.

7. No, the graph cannot represent a normal density function because it is not symmetric.

8. Yes, the graph can represent a normal density function.

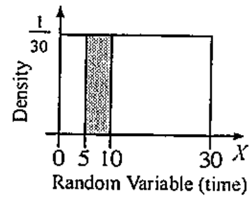
9. No, the graph cannot represent a normal density function because it crosses below the horizontal axis. That is, it is not always greater than or equal to 0.

10. No, the graph cannot represent a normal density function because it does not approach the horizontal axis as X increases (and decreases) without bound.

11. Yes, the graph can represent a normal density function.

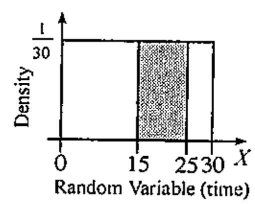
12. No, the graph cannot represent a normal density function because it is not bell-shaped.

13. The figure presents the graph of the density function with the area we wish to find shaded.



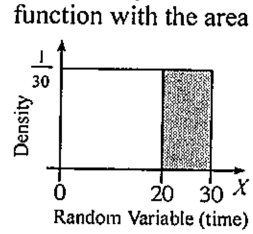
The width of the rectangle is $10-5=5$ and the height is $\frac{1}{30}$. Thus, the area between 5 and 10 is $5\left(\frac{1}{30}\right)=\frac{1}{6}$. The probability that the friend is between 5 and 10 minutes late is $\frac{1}{6}$.

14. The figure presents the graph of the density function with the area we wish to find shaded.



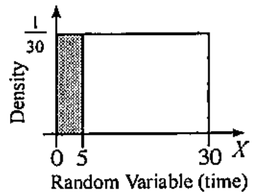
The width of the rectangle is $25-15=10$ and the height is $\frac{1}{30}$. Thus, the area between 15 and 25 is $10\left(\frac{1}{30}\right)=\frac{1}{3}$. The probability that the friend is between 15 and 25 minutes late is $\frac{1}{3}$.

15. The figure presents the graph of the density function with the area we wish to find shaded.



The width of the rectangle is $30-20=10$ and the height is $\frac{1}{30}$. Thus, the area between 20 and 30 is $10\left(\frac{1}{30}\right)=\frac{1}{3}$. The probability that the friend is at least 20 minutes late is $\frac{1}{3}$.

16. The figure presents the graph of the density function with the area we wish to find shaded.



The width of the rectangle is $5-0=5$ and the height is $\frac{1}{30}$. Thus, the area between 0 and 5 is $5\left(\frac{1}{30}\right)=\frac{1}{6}$. The probability that the friend is no more than 5 minutes late is $\frac{1}{6}$.

17. (a)

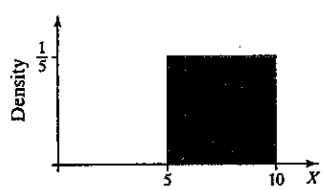
(b) $P(0 \le X \le 0.2)=1(0.2-0)=0.2$

(c) $P(0.25 \le X \le 0.6)=1(0.6-0.25)=0.35$

(d) $P(X \ge 0.95)=1(1-0.95)=0.05$

(e) Answers will vary.

18. (a)



(b) $P(6 \le X \le 8)=\frac{1}{5}(8-6)=\frac{2}{5}=0.4$

(c) $P(5 \le X \le 8)=\frac{1}{5}(8-5)=\frac{3}{5}=0.6$

(d) $P(X \le 6)=\frac{1}{5}(6-5)=\frac{1}{5}=0.2$

19. The histogram is symmetrical and bell-shaped, so a normal distribution can be used as a model for the variable.

20. The histogram is skewed to the right, so normal distribution cannot be used as a model for the variable.

21. The histogram is skewed to the right, so normal distribution cannot be used as a model for the variable.

22. The histogram is symmetrical and bell-shaped, so a normal distribution can be used as a model for the variable.

23. Graph A matches $\mu=10$ and $\sigma=2$, and graph B matches $\mu=10$ and $\sigma=3$. We can tell because a higher standard deviation makes the graph lower and more spread out.

24. Graph A matches $\mu=8$ and $\sigma=2$, and graph B matches $\mu=14$ and $\sigma=2$. We can tell because a larger mean shifts the graph to the right.

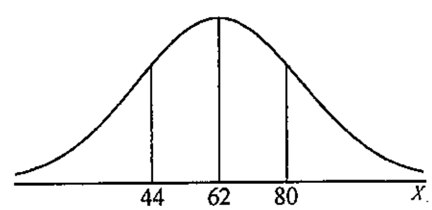
25. The center is at 2, so $\mu=2$. The distance to the inflection points is ± 3, so $\sigma=3$.

26. The center is at 5, so $\mu=5$. The distance to the inflection points is ± 2, so $\sigma=2$.

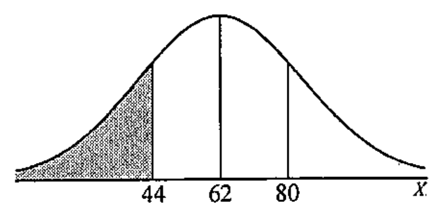
27. The center is at 100, so $\mu=100$. The distance to the inflection points is ± 15, so $\sigma=15$.

28. The center is at 530, so $\mu=530$. The distance to the inflection points is ± 100, so $\sigma=100$.

29. (a)



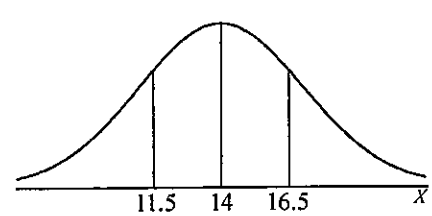
(b)



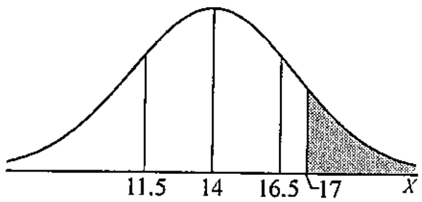
(c) Interpretation 1: 15.87% of the cell phone plans in the United States are less than \$44.00 per month.

Interpretation 2: The probability is 0.1587 that a randomly selected cell phone plan in the United States is less than \$44.00 per month.

30. (a)



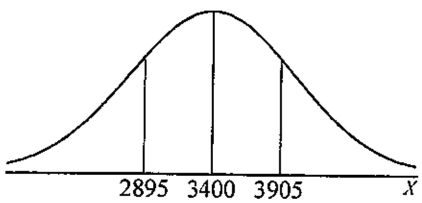
(b)



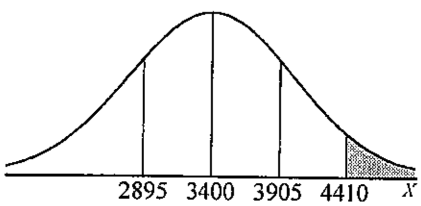
(c) Interpretation 1: 11.51% of all refrigerators are kept for more than 17 years.

Interpretation 2: The probability is 0.1151 that a randomly selected refrigerator is more than 17 years old.

31. (a)



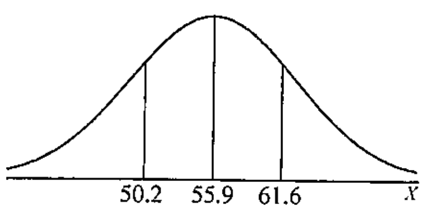
(b)



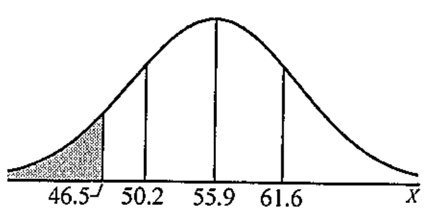
(c) Interpretation 1: 2.28% of all full-term babies has a birth weight of at least 4410 grams.

Interpretation 2: The probability is 0.0228 that the birth weight of a randomly chosen full-term baby is at least 4410 grams.

32. (a)



(b)



(c) Interpretation 1: The proportion of 10-year-old males who are less than 46.5 inches tall is 0.0496.

Interpretation 2: The probability is 0.0496 that a randomly selected 10-year-old male is less than 46.5 inches tall.

33. (a) Interpretation 1: The proportion of human pregnancies that last more than 280 days is 0.1908.

Interpretation 2: The probability is 0.1908 that a randomly selected human pregnancy lasts more than 280 days.

(b) Interpretation 1: The proportion of human pregnancies that last between 230 and 260 days is 0.3416.

Interpretation 2: The probability is 0.3416 that a randomly selected human pregnancy lasts between 230 and 260 days.

34. (a) Interpretation 1: The proportion of times that Elena gets more than 26 miles per gallon is 0.3309.

Interpretation 2: The probability is 0.3309 that a randomly selected fill-up yields at least 26 miles per gallon is 0.3309.

(b) Interpretation 1: The proportion of times that Elena gets between 18 and 21 miles per gallon is 0.1107.

Interpretation 2: The probability is 0.1107 that a randomly selected fill-up yields between 18 and 21 miles per gallon.

35. (a) $z_1 = \frac{x_1 - \mu}{\sigma} = \frac{8-10}{3} = -\frac{2}{3} \approx -0.67$

(b) $z_2 = \frac{x_2 - \mu}{\sigma} = \frac{12-10}{3} = \frac{2}{3} \approx 0.67$

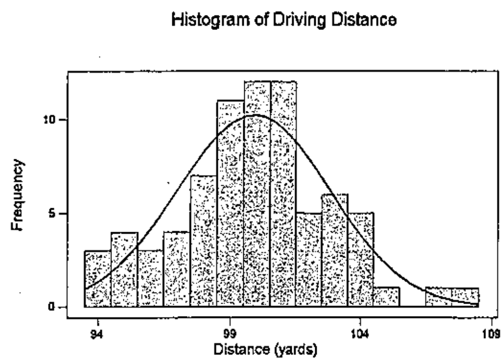
(c) The area between z_1 and z_2 is also 0.495.

36. (a) $z_1 = \frac{x_1 - \mu}{\sigma} = \frac{18-25}{6} = -\frac{7}{6} \approx -1.17$

(b) $z_2 = \frac{x_2 - \mu}{\sigma} = \frac{30-25}{6} = \frac{5}{6} \approx 0.83$

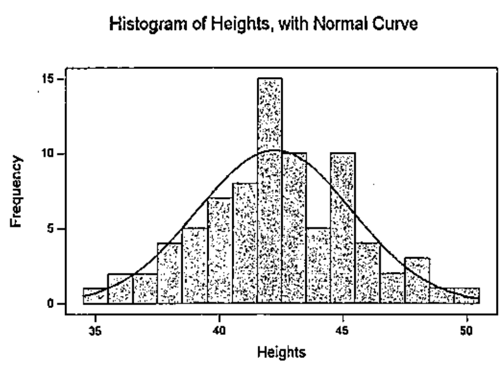
(c) The area between z_1 and z_2 is also 0.6760.

37. (a), (b)



(c) The normal density function appears to describe the distance Michael hits a pitching wedge fairly accurately. Looking at the graph, the normal curve is a fairly good approximation to the histogram.

38. (a), (b)



(c) The normal density function appears to describe the heights of 5-year-old females fairly accurately. Looking at the graph, the normal curve is a fairly good approximation to the histogram.

Section 7.2

1. The standard normal curve has the following properties:

(1) It is symmetric about its mean $\mu = 0$ and has standard deviation $\sigma = 1$.

(2) Its highest point occurs at $\mu = 0$.

(3) It has inflection points at -1 and 1.

(4) The area under the curve is 1.

(5) The area under the curve to the right of $\mu = 0$ equals the area under the curve to the left of $\mu = 0$. Both equal 0.5.

(6) As Z increases, the graph approaches, but never equals, zero. As Z decreases, the graph approaches, but never equals, zero.

(7) It satisfies the Empirical Rule: Approximately 68% of the area under the standard normal curve is between -1 and 1. Approximately 95% of the area under the standard normal curve is between -2 and 2. Approximately 99.7% of the area under the standard normal curve is between -3 and 3.

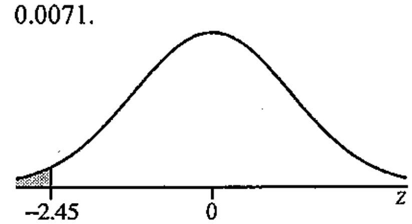
2. Since the total area under the normal curve is 1, the area to the right of $z = 1.20$ is $1 - 0.8849 = 0.1151$.

3. False. Although the area is very close to 1 (and for all practical purposes we may assume it is approximately 1), it is not exactly 1 because the normal curve continues indefinitely to the right.

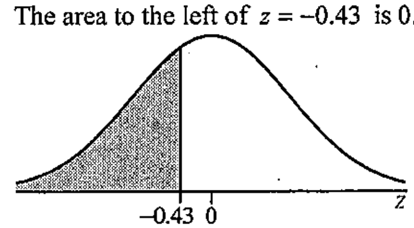
4. Since a normal random variable is a continuous random variable, the area "under" any specific value of Z, such as $z = -1.30$ is zero. Thus, $P(Z < -1.30) = P(Z \le -1.30)$. That is, the area strictly to the left of $z = -1.30$ is the same as the area up to and including $z = -1.30$.

5. The standard normal table (Table V) gives the area to the left of the z-score. Thus, we look up each z-score and read the corresponding area. We can also use technology. The areas are:

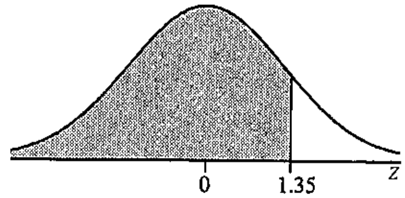
(a) The area to the left of $z = -2.45$ is 0.0071.



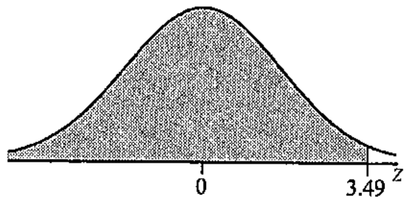
(b) The area to the left of $z = -0.43$ is 0.3336.



(c) The area to the left of $z = 1.35$ is 0.9115.

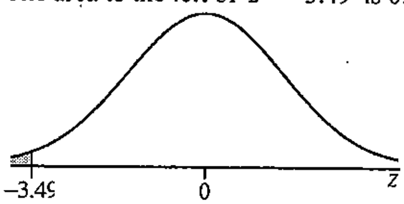


(d) The area to the left of $z = 3.49$ is 0.9998.

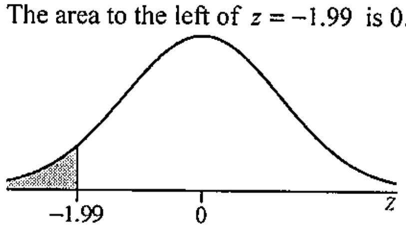


6. The standard normal table (Table V) gives the area to the left of the z-score. Thus, we look up each z-score and read the corresponding area. We can also use technology. The areas are:

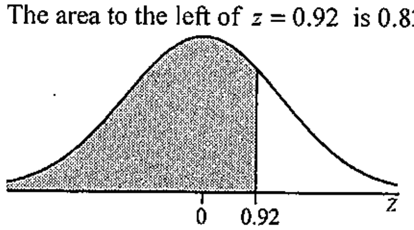
(a) The area to the left of $z = -3.49$ is 0.0002.



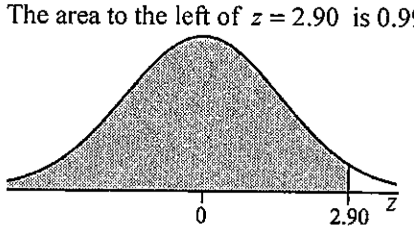
(b) The area to the left of $z = -1.99$ is 0.0233.



(c) The area to the left of $z = 0.92$ is 0.8212.

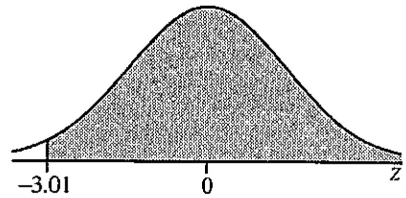


(d) The area to the left of $z = 2.90$ is 0.9981.

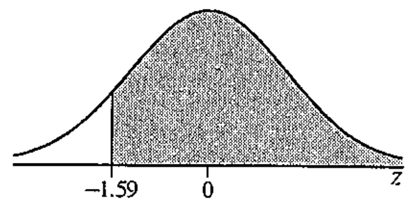


7. The standard normal table (Table V) gives the area to the left of the z-score. Thus, we look up each z-score and read the corresponding area from the table. The area to the right is one minus the area to the left. We can also use technology to find the area. The areas are:

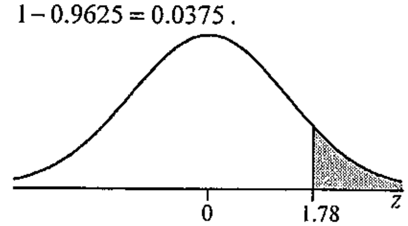
(a) The area to the right of $z = -3.01$ is $1 - 0.0013 = 0.9987$.



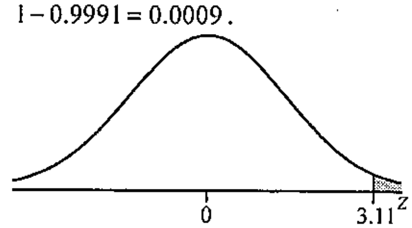
(b) The area to the right of $z = -1.59$ is $1 - 0.0559 = 0.9441$.



(c) The area to the right of $z = 1.78$ is $1 - 0.9625 = 0.0375$.

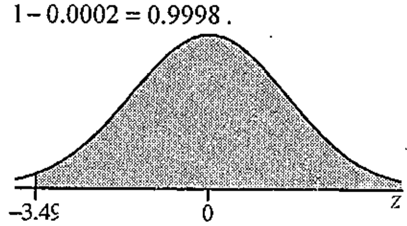


(d) The area to the right of $z = 3.11$ is $1 - 0.9991 = 0.0009$.

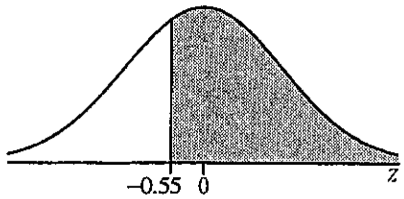


8. The standard normal table (Table V) gives the area to the left of the z-score. Thus, we look up each z-score and read the corresponding area from the table. The area to the right is one minus the area to the left. We can also use technology to find the area. The areas are:

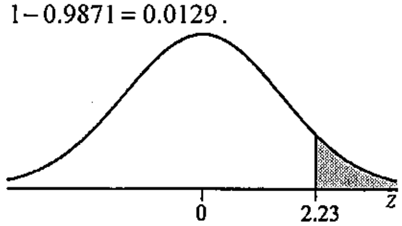
(a) The area to the right of $z = -3.49$ is $1 - 0.0002 = 0.9998$.



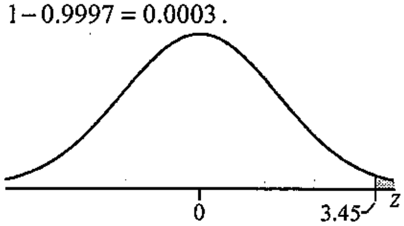
(b) The area to the right of $z=-0.55$ is $1-0.2912=0.7088$.



(c) The area to the right of $z=2.23$ is $1-0.9871=0.0129$.

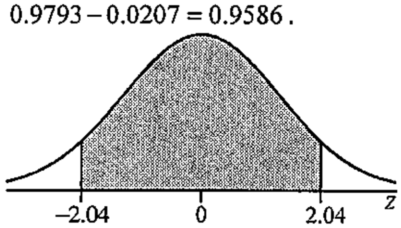


(d) The area to the right of $z=3.45$ is $1-0.9997=0.0003$.

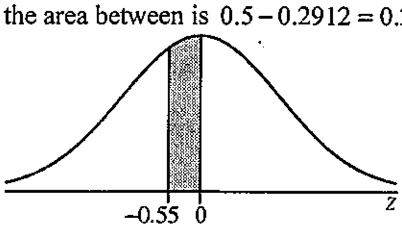


9. To find the area between two z-scores using the standard normal table (Table V), we look up the area to the left of each z-score and then we find the difference between these two. We can also use technology to find the area. The areas are:

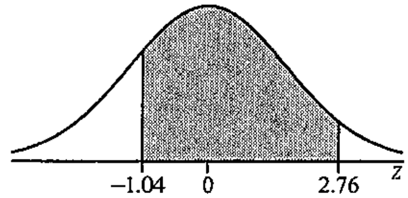
(a) The area to the left of $z=-2.04$ is 0.0207, and the area to the left of $z=2.04$ is 0.9793. So, the area between is $0.9793-0.0207=0.9586$.



(b) The area to the left of $z=-0.55$ is 0.2912, and the area to the left of $z=0$ is 0.5. So, the area between is $0.5-0.2912=0.2088$.

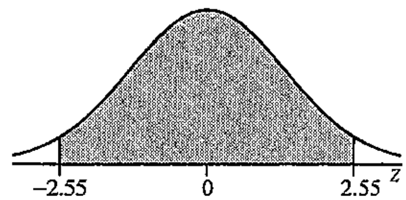


(c) The area to the left of $z=-1.04$ is 0.1492, and the area to the left of $z=2.76$ is 0.9971. So, the area between is $0.9971-0.1492=0.8479$

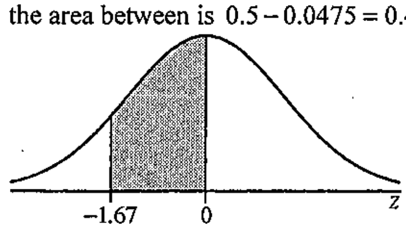


10. To find the area between two z-scores using the standard normal table (Table V), we look up the area to the left of each z-score and then we find the difference between these two. We can also use technology to find the area. The areas are:

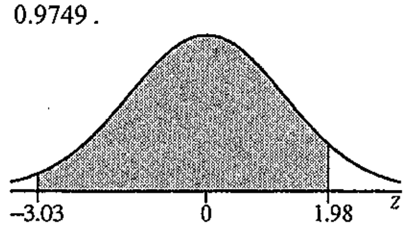
(a) The area to the left of $z=-2.55$ is 0.0054, and the area to the left of $z=2.55$ is 0.9946. So, the area between is $0.9946-0.0054=0.9892$.



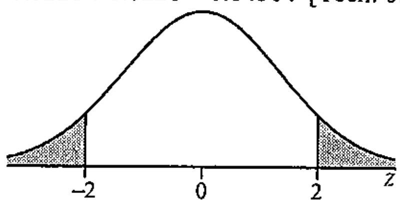
(b) The area to the left of $z=-1.67$ is 0.0475, and the area to the left of $z=0$ is 0.5. So, the area between is $0.5-0.0475=0.4525$.



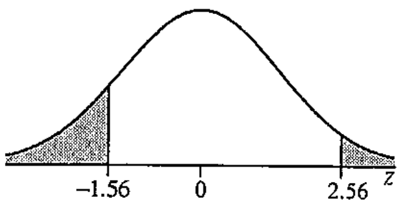
(c) The area to the left of $z=-3.03$ is 0.0012, and the area to the left of $z=1.98$ is 0.9761. So, the area between is $0.9761-0.0012=$ 0.9749.



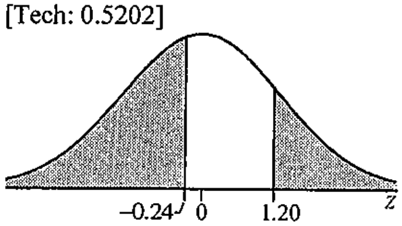
11. (a) The area to the left of $z = -2$ is 0.0228, and the area to the right of $z = 2$ is $1 - 0.9772 = 0.0228$. So, the total area is $0.0228 + 0.0228 = 0.0456$. [Tech: 0.0455]



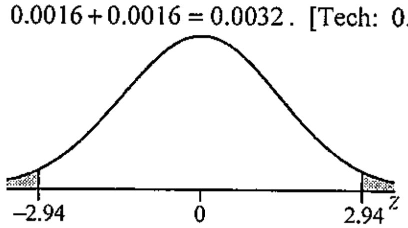
(b) The area to the left of $z = -1.56$ is 0.0594, and the area to the right of $z = 2.56$ is $1 - 0.9948 = 0.0052$. So, the total area is $0.0594 + 0.0052 = 0.0646$.



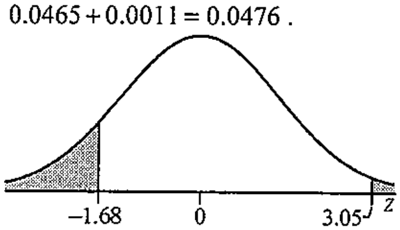
(c) The area to the left of $z = -0.24$ is 0.4052, and the area to the right of $z = 1.20$ is $1 - 0.8849 = 0.1151$. So, the total area is $0.4052 + 0.1151 = 0.5203$. [Tech: 0.5202]



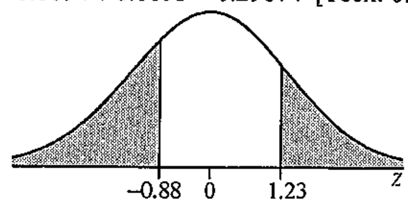
12. (a) The area to the left of $z = -2.94$ is 0.0016, and the area to the right of $z = 2.94$ is $1 - 0.9984 = 0.0016$. So, the total area is $0.0016 + 0.0016 = 0.0032$. [Tech: 0.0033]



(b) The area to the left of $z = -1.68$ is 0.0465, and the area to the right of $z = 3.05$ is $1 - 0.9989 = 0.0011$. So, the total area is $0.0465 + 0.0011 = 0.0476$.



(c) The area to the left of $z = -0.88$ is 0.1894, and the area to the right of $z = 1.23$ is $1 - 0.8907 = 0.1093$. So, the total area is $0.1894 + 0.1093 = 0.2987$. [Tech: 0.2988]



13. (a) The area to the left of $z = -1.34$ is 0.0901, and the area to the left of $z = 2.01$ is 0.9778. So, the area between is $0.9778 - 0.0901 = 0.8877$.

(b) The area to the left of $z = -1.96$ is 0.0250, and the area to the right of $z = 1.96$ is $1 - 0.9750 = 0.0250$. So, the total area is $0.0250 + 0.0250 = 0.0500$.

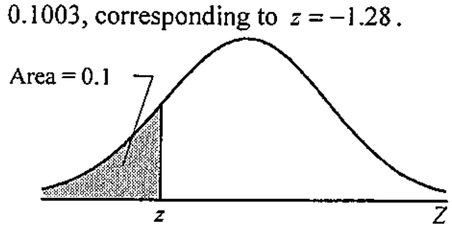
(c) The area to the left of $z = -2.33$ is 0.0099, and the area to the left of $z = 2.33$ is 0.9901. So, the area between $z = -2.33$ and $z = 2.33$ is $0.9901 - 0.0099 = 0.9802$.

14. (a) The area to the left of $z = -2.33$ is 0.0099, and the area to the right of $z = 2.33$ is $1 - 0.9901 = 0.0099$. So, the total area is $0.0099 + 0.0099 = 0.0198$.

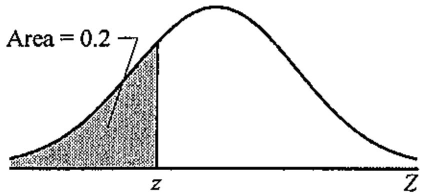
(b) The area to the left of $z = -1.12$ is 0.1314, and the area to the left of $z = 0$ is 0.5. So, the area between $z = -1.12$ and $z = 0$ is $0.5 - 0.1314 = 0.3686$.

(c) The area to the left of $z = -1.96$ is 0.0250, and the area to the left of $z = 1.96$ is 0.9750. So, the total area is $0.9750 - 0.0250 = 0.9500$.

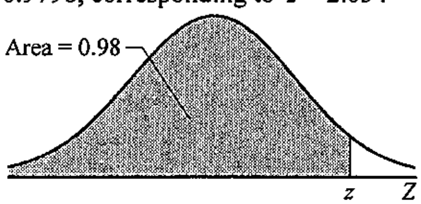
15. The area in the interior of the standard normal table (Table V) that is closest to 0.1000 is 0.1003, corresponding to $z = -1.28$.



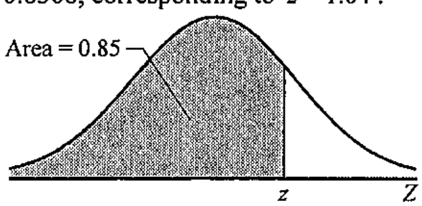
16. The area in the interior of the standard normal table (Table V) that is closest to 0.2000 is 0.2005, corresponding to $z = -0.84$.



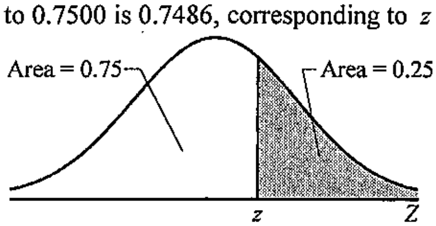
17. The area in the interior of the standard normal table (Table V) that is closest to 0.9800 is 0.9798, corresponding to $z = 2.05$.



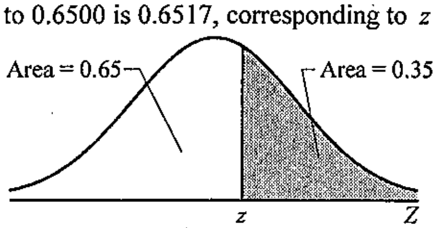
18. The area in the interior of the standard normal table (Table V) that is closest to 0.8500 is 0.8508, corresponding to $z = 1.04$.



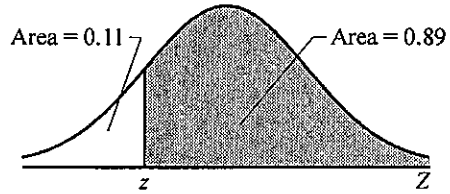
19. The area to the left of the unknown z-score is $1 - 0.25 = 0.75$. The area in the interior of the standard normal table (Table V) that is closest to 0.7500 is 0.7486, corresponding to $z = 0.67$.



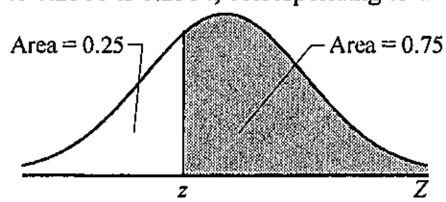
20. The area to the left of the unknown z-score is $1 - 0.35 = 0.65$. The area in the interior of the standard normal table (Table V) that is closest to 0.6500 is 0.6517, corresponding to $z = 0.39$.



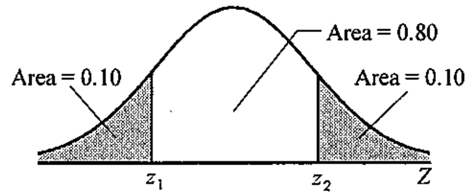
21. The area to the left of the unknown z-score is $1 - 0.89 = 0.11$. The area in the interior of the standard normal table (Table V) that is closest to 0.1100 is 0.1093, corresponding to $z = -1.23$.



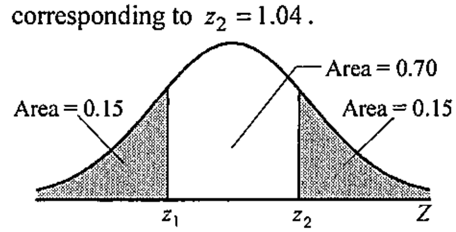
22. The area to the left of the unknown z-score is $1 - 0.75 = 0.25$. The area in the interior of the standard normal table (Table V) that is closest to 0.2500 is 0.2514, corresponding to $z = -0.67$.



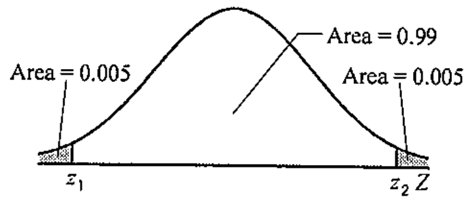
23. The z-scores for the middle 80% are the z-scores for the top and bottom 10%. The area to the left of z_1 is 0.10, and the area to the left of z_2 is 0.90. The area in the interior of the standard normal table (Table V) that is closest to 0.1000 is 0.1003, corresponding to $z_1 = -1.28$. The area in the interior of the standard normal table (Table V) that is closest to 0.9000 is 0.8997, corresponding to $z_2 = 1.28$.



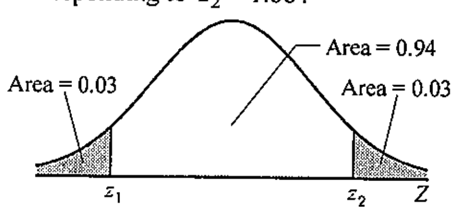
24. The z-scores for the middle 70% are the z-scores for the top and bottom 15%. The area to the left of z_1 is 0.15, and the area to the left of z_2 is 0.85. The area in the interior of the standard normal table (Table V) that is closest to 0.1500 is 0.1492, corresponding to $z_1 = -1.04$. The area in the interior of the standard normal table (Table V) that is closest to 0.8500 is 0.8508, corresponding to $z_2 = 1.04$.



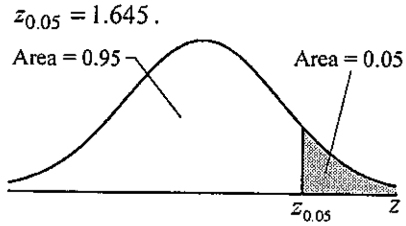
25. The z-scores for the middle 99% are the z-scores for the top and bottom 0.5%. The area to the left of z_1 is 0.005, and the area to the left of z_2 is 0.995. The areas in the interior of the standard normal table (Table V) that are closest to 0.0050 are 0.0049 and 0.0051. So, we use the average of their corresponding z-scores: -2.58 and -2.57, respectively. This gives $z_1 = -2.575$. The areas in the interior of the standard normal table (Table V) that are closest to 0.9950 are 0.9949 and 0.9951. So, we use the average of their corresponding z-scores: 2.57 and 2.58, respectively. This gives $z_2 = 2.575$.



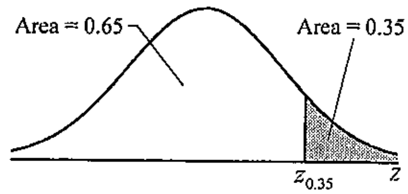
26. The z-scores for the middle 94% are the z-scores for the top and bottom 3%. The area to the left of z_1 is 0.03, and the area to the left of z_2 is 0.97. The area in the interior of the standard normal table (Table V) that is closest to 0.0300 is 0.0301, corresponding to $z_1 = -1.88$. The area in the interior of the standard normal table (Table V) that is closest to 0.9300 is 0.9699, corresponding to $z_2 = 1.88$.



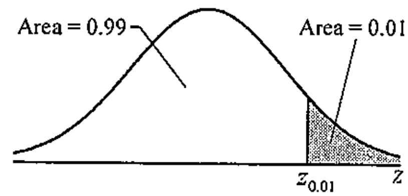
27. The area to the right of the unknown z-score is 0.05, so the area to the left is $1 - 0.05 = 0.9500$. From the interior of the standard normal table (Table V), we find that the z-scores 1.64 and 1.65 have corresponding areas of 0.9495 and 0.9505, respectively, which are equally close to 0.95. So, we average the two z-scores obtaining $z_{0.05} = 1.645$.



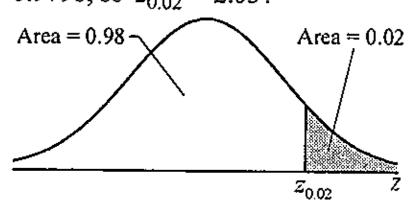
28. The area to the right of the unknown z-score is 0.35, so the area to the left is $1 - 0.35 = 0.65$. The area in the interior of the standard normal table (Table V) that is closest to 0.6500 is 0.6517, so $z_{0.35} = 0.39$.



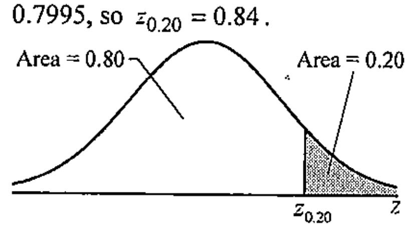
29. The area to the right of the unknown z-score is 0.01, so the area to the left is $1 - 0.01 = 0.99$. The area in the interior of the standard normal table (Table V) that is closest to 0.9900 is 0.9901, so $z_{0.01} = 2.33$.



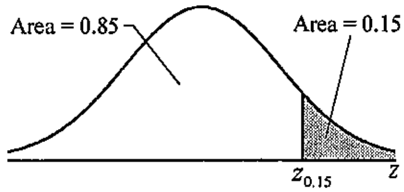
30. The area to the right of the unknown z-score is 0.02, so the area to the left is $1 - 0.02 = 0.98$. The area in the interior of the standard normal table (Table V) that is closest to 0.9800 is 0.9798, so $z_{0.02} = 2.05$.



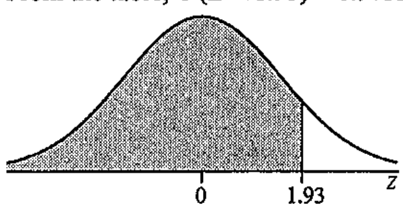
31. The area to the right of the unknown z-score is 0.20, so the area to the left is $1 - 0.20 = 0.80$. The area in the interior of the standard normal table (Table V) that is closest to 0.8000 is 0.7995, so $z_{0.20} = 0.84$.



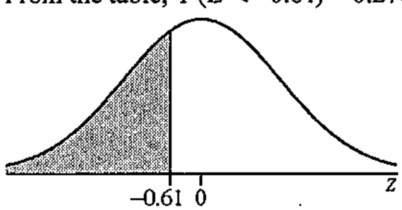
32. The area to the right of the unknown z-score is 0.15, so the area to the left is $1 - 0.15 = 0.85$. The area in the interior of the standard normal table (Table V) that is closest to 0.8500 is 0.8508, so $z_{0.15} = 1.04$.



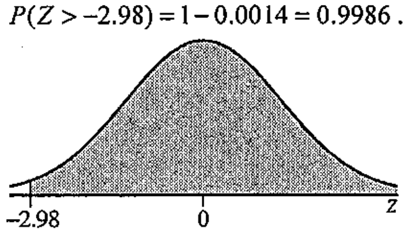
33. From the table, $P(Z < 1.93) = 0.9732$.



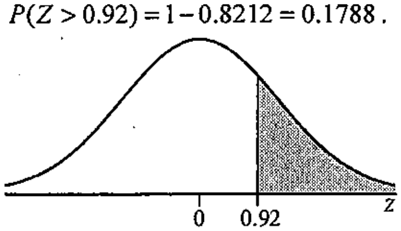
34. From the table, $P(Z < -0.61) = 0.2709$.



35. From the table, $P(Z < -2.98) = 0.0014$, so $P(Z > -2.98) = 1 - 0.0014 = 0.9986$.

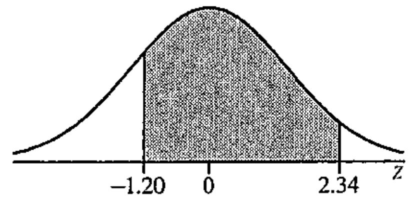


36. From the table, $P(Z < 0.92) = 0.8212$, so $P(Z > 0.92) = 1 - 0.8212 = 0.1788$.



37. From the table, $P(Z < -1.20) = 0.1151$ and $P(Z < 2.34) = 0.9904$. So,

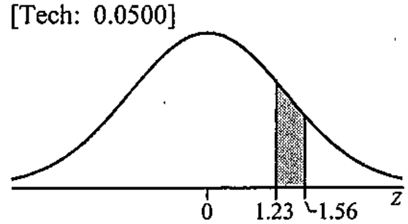
$$P(-1.20 \le Z < 2.34) = 0.9904 - 0.1151 = 0.8753$$



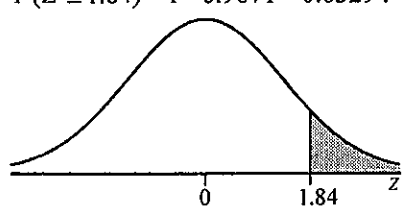
38. From the table, $P(Z < 1.23) = 0.8907$ and $P(Z < 1.56) = 0.9406$. So,

$$P(1.23 < Z \le 1.56) = 0.9406 - 0.8907 = 0.0499$$

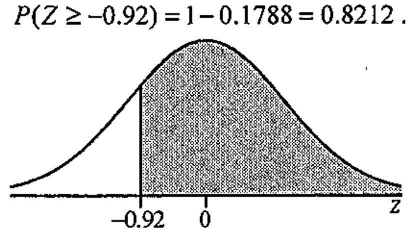
[Tech: 0.0500]



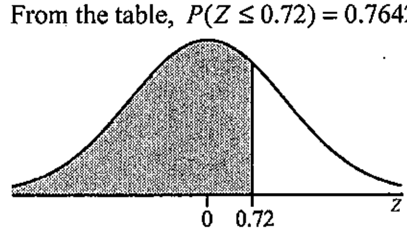
39. From the table, $P(Z < 1.84) = 0.9671$, so $P(Z \ge 1.84) = 1 - 0.9671 = 0.0329$.



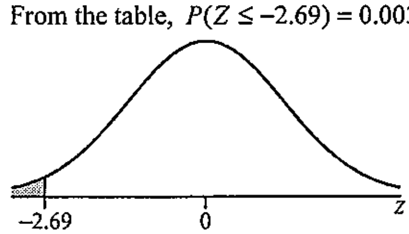
40. From the table, $P(Z < -0.92) = 0.1788$, so $P(Z \ge -0.92) = 1 - 0.1788 = 0.8212$.



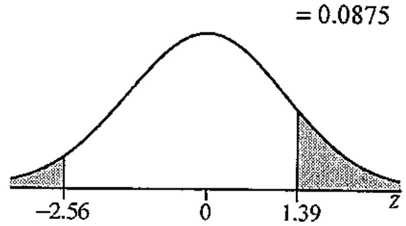
41. From the table, $P(Z \le 0.72) = 0.7642$



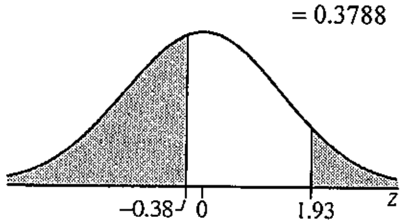
42. From the table, $P(Z \le -2.69) = 0.0036$.



43. From the table, $P(Z < -2.56) = 0.0052$.
Also, $P(Z \le 1.39) = 0.9177$, which means
$P(Z > 1.39) = 1 - 0.9177 = 0.0823$. So,
$$P(Z < -2.56 \text{ or } Z > 1.39) = 0.0052 + 0.0823 = 0.0875$$



44. From the table, $P(Z < -0.38) = 0.3520$.
Also, $P(Z < 1.93) = 0.9732$, which means
$P(Z > 1.93) = 1 - 0.9732 = 0.0268$. So,
$$P(Z < -0.38 \text{ or } Z > 1.93) = 0.3520 + 0.0268 = 0.3788$$



45. From the table, $P(Z < -1.00) = 0.1587$ and
$P(Z < 1.00) = 0.8413$, so
$$P(-1 < Z < 1) = 0.8413 - 0.1587 = 0.6826$$
[Tech: 0.6827]
So, approximately 68% of the data lies within 1 standard deviation of the mean.

Similarly, $P(Z < -2.00) = 0.0228$ and
$P(Z < 2.00) = 0.9772$, so
$$P(-2 < Z < 2) = 0.9772 - 0.0228 = 0.9544$$
[Tech: 0.9545]
So, approximately 95% of the data lies within 2 standard deviations of the mean.
Likewise, $P(Z < -3.00) = 0.0013$ and
$P(Z < 3.00) = 0.9987$, so
$$P(-3 < Z < 3) = 0.9987 - 0.0013 = 0.9974$$
[Tech: 0.9973]
So, approximately 99.7% of the data lies within 3 standard deviations of the mean.

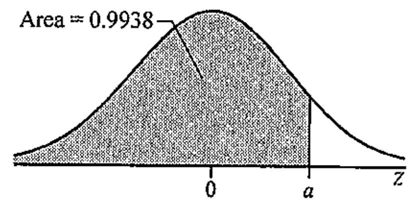
46. By symmetry, the area to the right of $z = 1.34$ is also 0.0901.

47. By symmetry, the area to the right of $z = 2.55$ is also 0.0054.

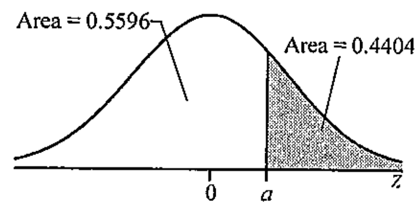
48. By symmetry, the area between $z = 0$ and $z = 1.50$ is also 0.4332.

49. By symmetry, the area between $z = 0.53$ and $z = 1.24$ is also 0.1906.

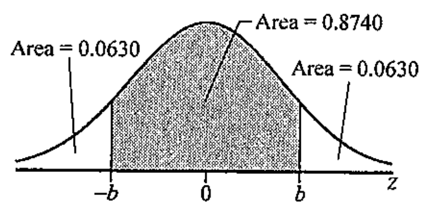
50. **(a)** From the table, the area 0.9938 corresponds to $z = 2.50$, so $P(Z < 2.50) = 0.9938$. So, $a = 2.50$.



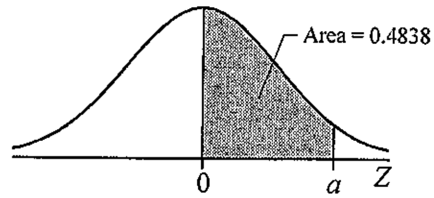
(b) If the area to the right of a is 0.4404, then the area to the left of a is $1 - 0.4404 = 0.5596$, which corresponds to $z = 0.15$. So, $P(Z \ge 0.15) = 0.4404$ and $a = 0.15$.



(c) If $P(-b < Z < b) = 0.8740$ then
$P(Z < -b) = \frac{1}{2}(1 - 0.8740) = 0.0630$.
From the table, this area corresponds to $z = -1.53$. By symmetry,
$P(-1.53 < Z < 1.53) = 0.8740$, so $b = 1.53$.



(d) If $P(0 < Z < a) = 0.4838$ then $P(Z < a) = 0.5 + 0.4838 = 0.9838$. From the table, this area corresponds to $z = 2.14$.

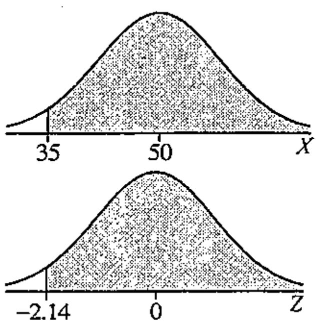


Section 7.3

1. To find the area under any normal curve:
   (1) Draw the curve and shade the desired area.
   (2) Convert the values of X to z-scores using the formula $z = \dfrac{x-\mu}{\sigma}$.
   (3) Draw a standard normal curve and shade the desired area.
   (4) Find the area under the standard normal curve. This area is equal to the area under the normal curve drawn in Step (1).

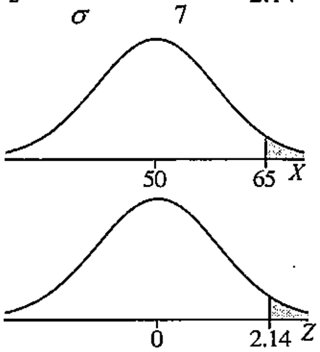
2. To find the score that corresponds to a given probability:
   (1) Draw a normal curve and shade the area corresponding to the probability that is given.
   (2) Use Table V (the standard normal table) to find the z-score that corresponds to the shaded area.
   (3) Obtain the normal value using the formula $x = \mu + z\sigma$.

3. $z = \dfrac{x-\mu}{\sigma} = \dfrac{35-50}{7} \approx -2.14$



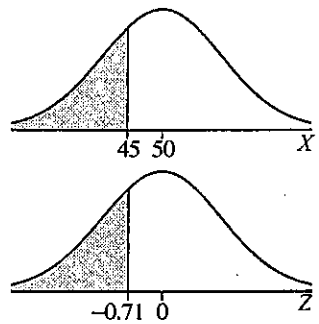
From Table V, the area to the left is 0.0162, so $P(X > 35) = 1 - 0.0162 = 0.9838$. [Tech: 0.9839]

4. $z = \dfrac{x-\mu}{\sigma} = \dfrac{65-50}{7} \approx 2.14$



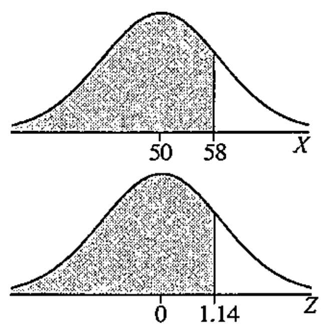
From Table V, the area to the left is 0.9838, so $P(X > 65) = 1 - 0.9838 = 0.0162$.
[Tech: 0.0161]

5. $z = \dfrac{x-\mu}{\sigma} = \dfrac{45-50}{7} \approx -0.71$



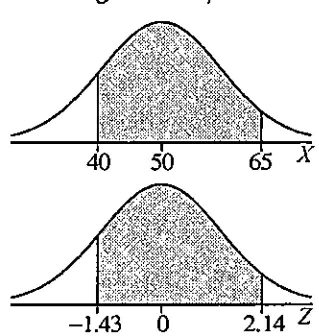
From Table V, the area to the left is 0.2389, so $P(X \le 45) = 0.2389$. [Tech: 0.2375]

6. $z = \dfrac{x-\mu}{\sigma} = \dfrac{58-50}{7} \approx 1.14$



From Table V, the area to the left is 0.8729, so $P(X \le 58) = 0.8729$. [Tech: 0.8735]

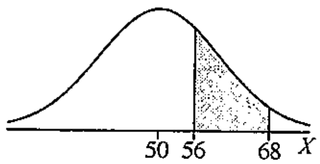
7. $z_1 = \dfrac{x_1-\mu}{\sigma} = \dfrac{40-50}{7} \approx -1.43$;
   $z_2 = \dfrac{x_2-\mu}{\sigma} = \dfrac{65-50}{7} \approx 2.14$

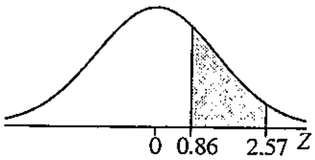


From Table V, the area to the left of $z_1 = -1.43$ is 0.0764 and the area to the left of $z_2 = 2.14$ is 0.9838, so $P(40 < X < 65) =$ $0.9838 - 0.0764 = 0.9074$. [Tech: 0.9074]

8. $z_1 = \dfrac{x_1 - \mu}{\sigma} = \dfrac{56-50}{7} \approx 0.86$;

$z_2 = \dfrac{x_1 - \mu}{\sigma} = \dfrac{68-50}{7} \approx 2.57$

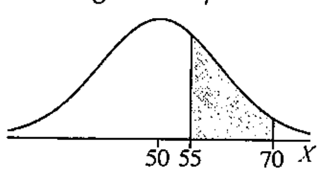


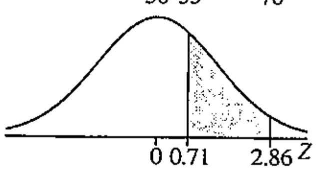


From Table V, the area to the left of $z_1 = 0.86$ is 0.8051 and the area to the left of $z_2 = 2.57$ is 0.9949, so $P(56 < X < 68) =$ $0.9949 - 0.8051 = 0.1898$. [Tech: 0.1906]

9. $z_1 = \dfrac{x_1 - \mu}{\sigma} = \dfrac{55-50}{7} \approx 0.71$;

$z_2 = \dfrac{x_2 - \mu}{\sigma} = \dfrac{70-50}{7} \approx 2.86$

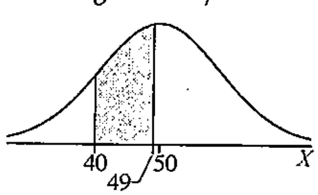


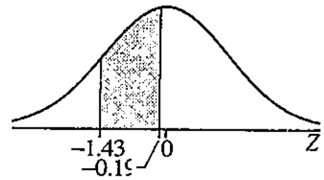


From Table V, the area to the left of $z_1 = 0.71$ is 0.7611 and the area to the left of $z_2 = 2.86$ is 0.9979, so $P(55 \le X \le 70) =$ $0.9979 - 0.7611 = 0.2368$. [Tech: 0.2354]

10. $z_1 = \dfrac{x_1 - \mu}{\sigma} = \dfrac{40-50}{7} \approx -1.43$;

$z_2 = \dfrac{x_2 - \mu}{\sigma} = \dfrac{49-50}{7} \approx -0.19$

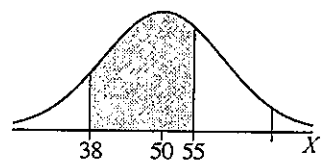


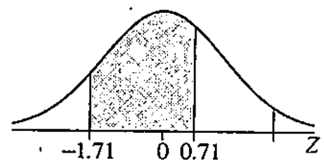


From Table V, the area to the left of $z_1 = -1.43$ is 0.0764 and the area to the left of $z_2 = -0.19$ is 0.4443, so $P(40 \le X < 49) =$ $0.4443 - 0.0764 = 0.3679$. [Tech: 0.3666]

11. $z_1 = \dfrac{x_1 - \mu}{\sigma} = \dfrac{38-50}{7} \approx -1.71$;

$z_2 = \dfrac{x_2 - \mu}{\sigma} = \dfrac{55-50}{7} \approx 0.71$

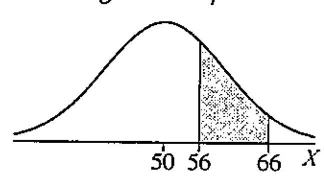


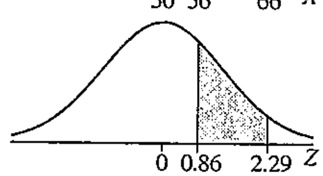


From Table V, the area to the left of $z_1 = -1.71$ is 0.0436 and the area to the left of $z_2 = 0.71$ is 0.7611, so $P(38 < X \le 55) = 0.7611 - 0.0436$ $= 0.7175$. [Tech: 0.7192]

12. $z_1 = \dfrac{x_1 - \mu}{\sigma} = \dfrac{56-50}{7} \approx 0.86$;

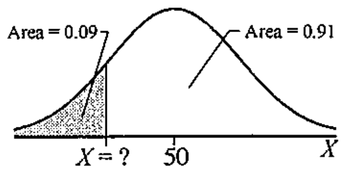
$z_2 = \dfrac{x_2 - \mu}{\sigma} = \dfrac{66-50}{7} \approx 2.29$





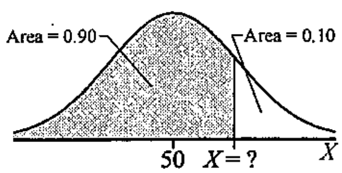
From Table V, the area to the left of $z_1 = 0.86$ is 0.8051 and the area to the left of $z_2 = 2.29$ is 0.9890, so $P(56 \le X < 66) = 0.9890 - 0.8051$ $= 0.1839$. [Tech: 0.1845]

13. The figure below shows the normal curve with the unknown value of X separating the bottom 9% of the distribution from the top 91% of the distribution.



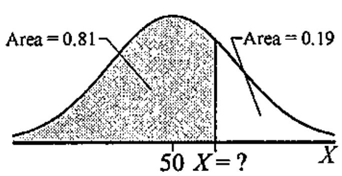
From Table V, the area closest to 0.09 is 0.0901. The corresponding z-score is -1.34. So, the 9[th] percentile for X is $x = \mu + z\sigma =$ $50 + (-1.34)(7) = 40.62$. [Tech: 40.61]

14. The figure below shows the normal curve with the unknown value of X separating the bottom 90% of the distribution from the top 10% of the distribution.



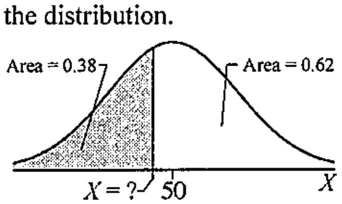
From Table V, the area closest to 0.90 is 0.8997. The corresponding z-score is 1.28. So, the 90[th] percentile for X is $x = \mu + z\sigma =$ $50 + 1.28(7) = 58.96$. [Tech: 58.97]

15. The figure below shows the normal curve with the unknown value of X separating the bottom 81% of the distribution from the top 19% of the distribution.



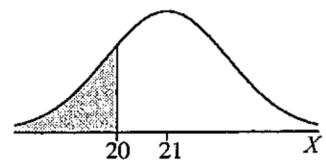
From Table V, the area closest to 0.81 is 0.8106. The corresponding z-score is 0.88. So, the 81[st] percentile for X is $x = \mu + z\sigma =$ $50 + 0.88(7) = 56.16$. [Tech: 56.15]

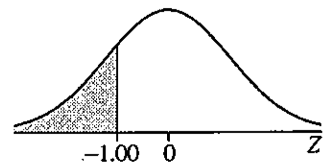
16. The figure below shows the normal curve with the unknown value of X separating the bottom 38% of the distribution from the top 62% of the distribution.



From Table V, the area closest to 0.38 is 0.3783. The corresponding z-score is -0.31. So, the 38[th] percentile for X is $x = \mu + z\sigma =$ $50 + (-0.31)(7) = 47.83$. [Tech: 47.86]

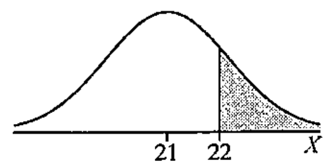
17. **(a)** $z = \dfrac{x-\mu}{\sigma} = \dfrac{20-21}{1} = -1.00$

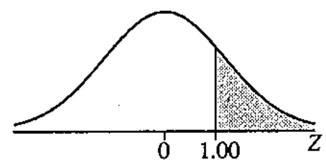




From Table V, the area to the left of $z = -1.00$ is 0.1587, so $P(X < 20) = 0.1587$.

(b) $z = \dfrac{x-\mu}{\sigma} = \dfrac{22-21}{1} = 1.00$

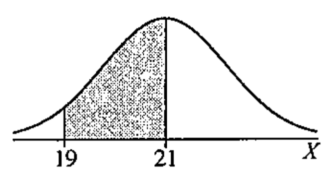


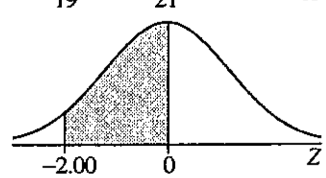


From Table V, the area to the left of $z = 1.00$ is 0.8413, so $P(X > 22) =$ $1 - 0.8413 = 0.1587$.

(c) $z_1 = \dfrac{x_1-\mu}{\sigma} = \dfrac{19-21}{1} = -2.00$;

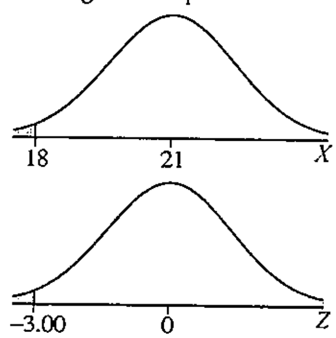
$z_2 = \dfrac{x_2-\mu}{\sigma} = \dfrac{21-21}{1} = 0$





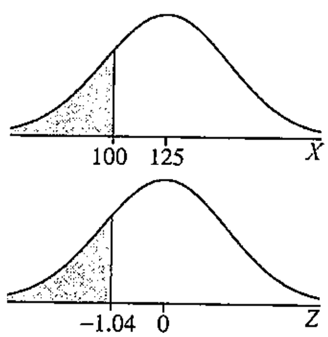
From Table V, the area to the left of $z_1 = -2.00$ is 0.0228 and the area to the left of $z_2 = 0$ is 0.5000, so $P(19 \le X \le 21) = 0.5000 - 0.0228 = 0.4772$.

(d) $z = \dfrac{x-\mu}{\sigma} = \dfrac{18-21}{1} = -3.00$



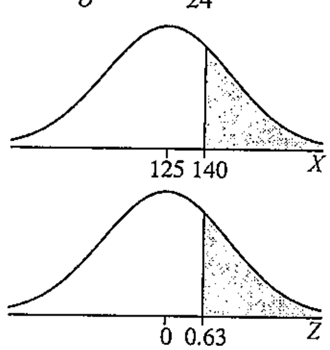
From Table V, the area to the left of $z = -3.00$ is 0.0013, so $P(X < 18) = 0.0013$.
Yes, it would be unusual for an egg to hatch in less than 18 days. Only about 1 egg in 1000 hatches in less than 18 days.

18. (a) $z = \dfrac{x-\mu}{\sigma} = \dfrac{100-125}{24} \approx -1.04$



From Table V, the area to the left of $z = -1.04$ is 0.1492, so $P(X < 100) = 0.1492$. [Tech: 0.1488]

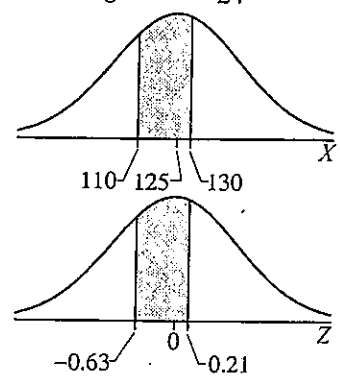
(b) $z = \dfrac{x-\mu}{\sigma} = \dfrac{140-125}{24} \approx 0.63$



From Table V, the area to the left of $z = 0.63$ is 0.7357, so $P(X > 140) = 1 - 0.7357 = 0.2643$. [Tech: 0.2660]

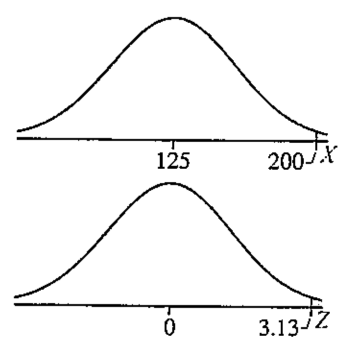
(c) $z_1 = \dfrac{x_1-\mu}{\sigma} = \dfrac{110-125}{24} \approx -0.63$;

$z_2 = \dfrac{x_2-\mu}{\sigma} = \dfrac{130-125}{24} \approx 0.21$



From Table V, the area to the left of $z_1 = -0.63$ is 0.2643 and the area to the left of $z_2 = 0.21$ is 0.5832, so $P(110 \le X \le 130) = 0.5832 - 0.2643 = 0.3189$ [Tech: 0.3165]

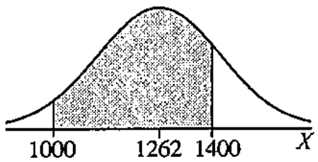
(d) $z = \dfrac{x-\mu}{\sigma} = \dfrac{200-125}{24} \approx 3.13$

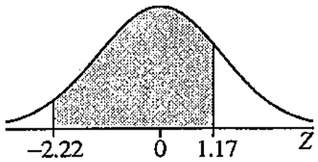


From Table V, the area to the left of $z = 3.13$ is 0.9991, so $P(X > 200) = 1 - 0.9991 = 0.0009$.

Yes, it would be unusual for a sixth-grade student to read more than 200 words per minute. Fewer than 1 in 1000 sixth-grade students read more than 200 words per minute.

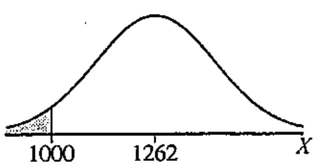
19. (a) $z_1 = \dfrac{x_1 - \mu}{\sigma} = \dfrac{1000 - 1262}{118} \approx -2.22$;
$z_2 = \dfrac{x_2 - \mu}{\sigma} = \dfrac{1400 - 1262}{118} \approx 1.17$

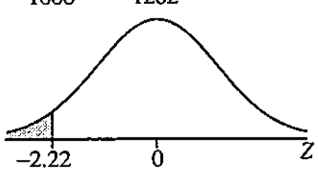




From Table V, the area to the left of $z_1 = -2.22$ is 0.0132 and the area to the left of $z_2 = 1.17$ is 0.8790, so $P(1000 \le X \le 1400) = 0.8790 - 0.0132 = 0.8658$. [Tech: 0.8657]

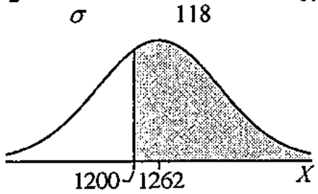
(b) $z = \dfrac{x - \mu}{\sigma} = \dfrac{1000 - 1262}{118} \approx -2.22$

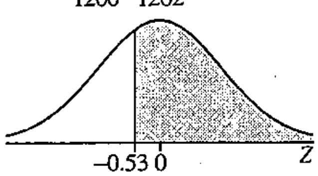




From Table V, the area to the left of $z = -2.22$ is 0.0132, so $P(X < 1000) = 0.0132$.

(c) $z = \dfrac{x - \mu}{\sigma} = \dfrac{1200 - 1262}{118} \approx -0.53$

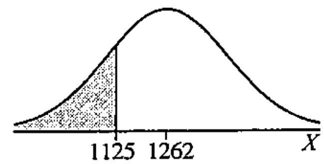


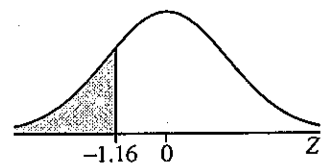


From Table V, the area to the left of $Z = -0.53$ is 0.2981, so $P(X > 1200) = 1 - 0.2981 = 0.7019$. [Tech: 0,7004]

So, the proportion of 18-ounce bags of Chip Ahoy! cookies that contains more than 1200 chocolate chips is 0.7019, or 70.19%.

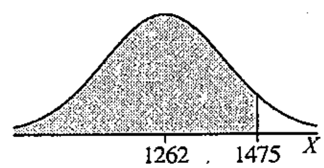
(d) $z = \dfrac{x - \mu}{\sigma} = \dfrac{1125 - 1262}{118} \approx -1.16$.

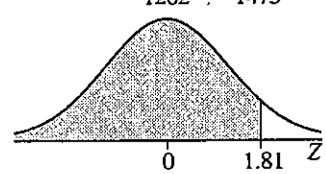




From Table V, the area to the left of $z = -1.16$ is 0.1230, so $P(X < 1125) = 0.1230$. [Tech: 0.1228] So, the proportion of 18-ounce bags of Chip Ahoy! cookies that contains less than 1125 chocolate chips is 0.1230, or 12.30%.

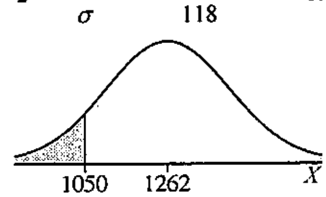
(e) $z = \dfrac{x - \mu}{\sigma} = \dfrac{1475 - 1262}{118} \approx 1.81$

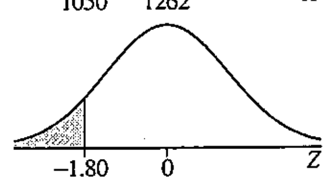




From Table V, the area to the left of $z = 1.81$ is 0.9649, so $P(X < 1475) = 0.9649$. [Tech: 0.9645] An 18-ounce bag of Chip Ahoy! Cookies that contains 1475 chocolate chips is at the 96th percentile.

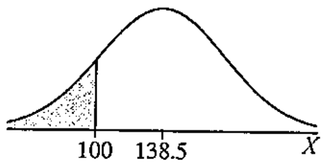
(f) $z = \dfrac{x - \mu}{\sigma} = \dfrac{1050 - 1262}{118} \approx -1.80$.

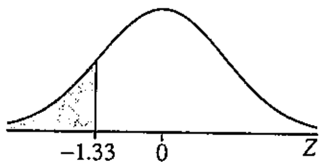




From Table V, the area to the left of $z=-1.80$ is 0.0359, so $P(X<1050)=0.0359$. [Tech: 0.0362] An 18-ounce bag of Chip Ahoy! cookies that contains 1050 chocolate chips is at the 4th percentile.

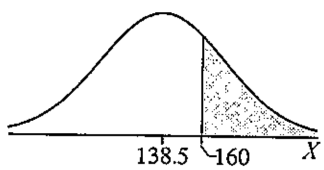
20. **(a)** $z=\dfrac{x-\mu}{\sigma}=\dfrac{100-138.5}{29}\approx -1.33$

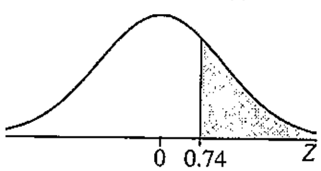




From Table V, the area to the left of $z=-1.33$ is 0.0918, so $P(X<100)=0.0918$. [Tech: 0.0922]

(b) $z=\dfrac{x-\mu}{\sigma}=\dfrac{160-138.5}{29}\approx 0.74$



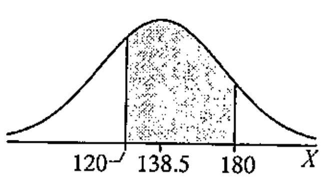


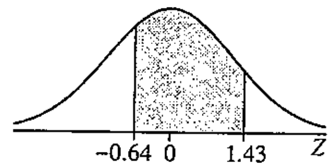
From Table V, the area to the left of $z=0.74$ is 0.7704, so $P(X>160)=1-0.7704=0.2296$. [Tech: 0.2292]

(c) Note that 2 minutes = 120 seconds and 3 minutes = 180 seconds.

$z_1=\dfrac{x_1-\mu}{\sigma}=\dfrac{120-138.5}{29}\approx -0.64$;

$z_2=\dfrac{x_2-\mu}{\sigma}=\dfrac{180-138.5}{29}\approx 1.43$

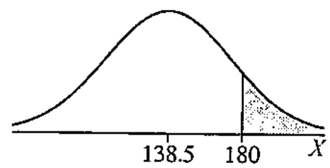


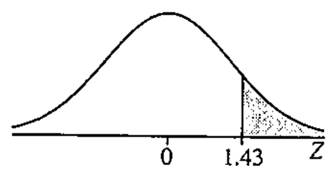


From Table V, the area to the left of $z_1=-0.64$ is 0.2611 and the area to the left of $z_2=1.43$ is 0.9236, so $P(120\le X\le 180)=0.9236-0.2611=0.6625$. [Tech: 0.6620]

(d) Note that 3 minutes = 180 seconds.

$z=\dfrac{x-\mu}{\sigma}=\dfrac{180-138.5}{29}\approx 1.43$

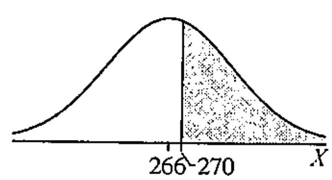


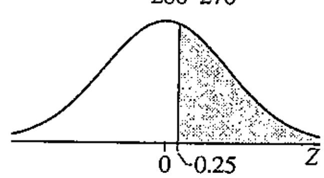


From Table V, the area to the left of $z=1.43$ is 0.9236, so $P(X\ge 180)=1-0.9236=0.0764$.

No, it would not be unusual for a car to spend more than 3 minutes (180 seconds) in Wendy's drive-through. About 8 cars in 100 will spend more than 3 minutes in the drive-through.

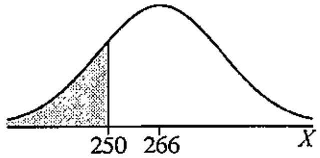
21. **(a)** $z=\dfrac{x-\mu}{\sigma}=\dfrac{270-266}{16}=0.25$

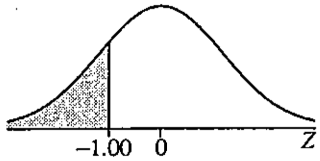




From Table V, the area to the left of $z=0.25$ is 0.5987, so $P(X>270)=1-0.5987=0.4013$. So, the proportion of human pregnancies that last more than 270 days is 0.4013.

(b) $z = \dfrac{x-\mu}{\sigma} = \dfrac{250-266}{16} = -1$

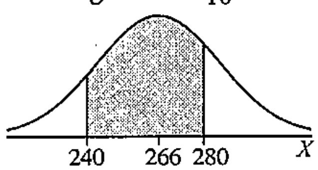


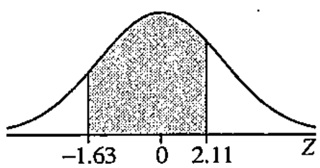


From Table V, the area to the left of $z = -1.00$ is 0.1587, so $P(X < 250) = 0.1587$. So, the proportion of human pregnancies that last less than 250 days is 0.1587, or 15.87%.

(c) $z_1 = \dfrac{x_1-\mu}{\sigma} = \dfrac{240-266}{16} \approx -1.63$;

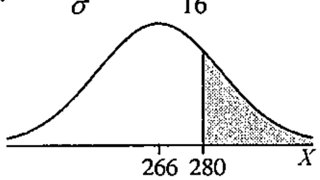
$z_2 = \dfrac{x_2-\mu}{\sigma} = \dfrac{280-266}{16} \approx 0.88$

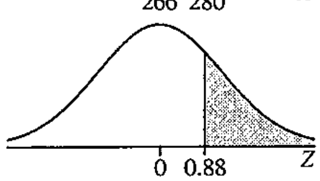




From Table V, the area to the left of $z_1 = -1.63$ is 0.0516 and the area to the left of $z_2 = 0.88$ is 0.8106, so $P(240 \le X \le 280) = 0.8106 - 0.0516 = 0.7590$. [Tech: 0.7571] So, the proportion of human pregnancies lasts between 240 and 280 days is 0.7590, or 75.90%.

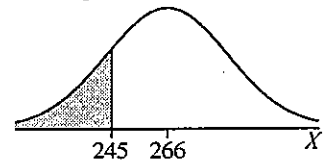
(d) $z = \dfrac{x-\mu}{\sigma} = \dfrac{280-266}{16} \approx 0.88$

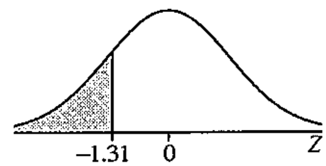




From Table V, the area to the left of $z = 0.88$ is 0.8106, so $P(X > 280) = 1 - 0.8106 = 0.1894$. [Tech: 0.1908]

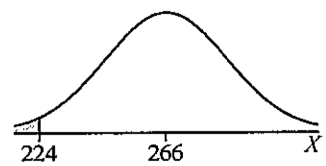
(e) $z = \dfrac{x-\mu}{\sigma} = \dfrac{245-266}{16} \approx -1.31$

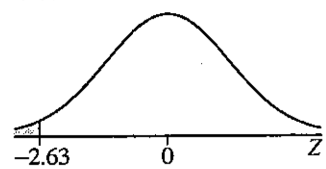




From Table V, the area to the left of $z = -1.31$ is 0.0951, so $P(X \le 245) = 0.0951$. [Tech: 0.0947]

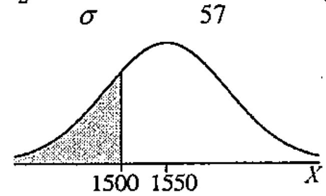
(f) $z = \dfrac{x-\mu}{\sigma} = \dfrac{224-266}{16} \approx -2.63$

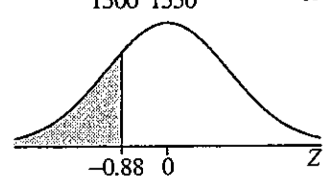




From Table V, the area to the left of $z = -2.63$ is 0.0043, so $P(X < 224) = 0.0043$.
Yes, "very preterm" babies are unusual. Only about 4 births in 1000 are "very preterm."

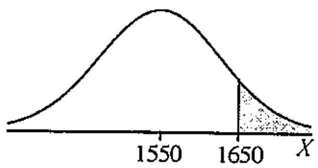
22. (a) $z = \dfrac{x-\mu}{\sigma} = \dfrac{1500-1550}{57} \approx -0.88$

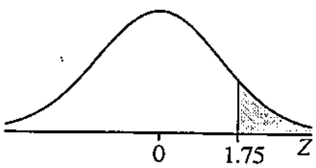




From Table V, the area to the left of $z = -0.88$ is 0.1894, so $P(X < 1500) =$ 0.1894. [Tech: 0.1902] So, the proportion of the light bulbs that will last less than the advertised time is 0.1894, or 18.94%.

(b) $z = \dfrac{x - \mu}{\sigma} = \dfrac{1650 - 1550}{57} \approx 1.75$

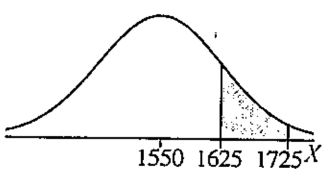


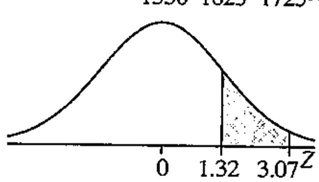


From Table V, the area to the left of $z = 1.75$ is 0.9599, so $P(X > 1650) =$ $1 - 0.9599 = 0.0401$. [Tech: 0.0397] So, the proportion of the light bulbs that will last longer than 1650 hours is 0.0401, or 4.01%.

(c) $z_1 = \dfrac{x_1 - \mu}{\sigma} = \dfrac{1625 - 1550}{57} \approx 1.32$;

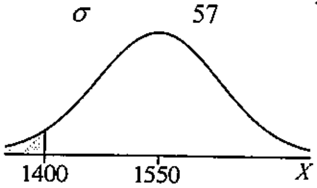
$z_2 = \dfrac{x_1 - \mu}{\sigma} = \dfrac{1725 - 1550}{57} \approx 3.07$.

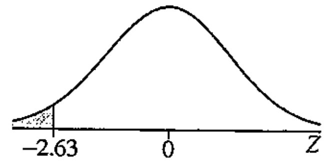




From Table V, the area to the left of $z_1 = 1.32$ is 0.9066 and the area to the left of $Z_2 = 3.07$ is 0.9989, so $P(1625 \le X \le 1725) = 0.9989 - 0.9066 =$ 0.0923. [Tech: 0.0931]

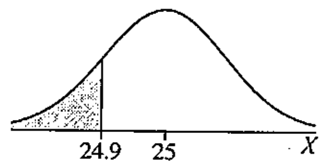
(d) $z = \dfrac{x - \mu}{\sigma} = \dfrac{1400 - 1550}{57} = -2.63$

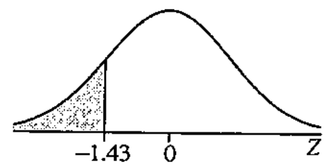




From Table V, the area to the left of $z = -2.63$ is 0.0043, so $P(X > 1400) =$ $1 - 0.0043 = 0.9957$. [Tech: 0.9958]

23. (a) $z = \dfrac{x - \mu}{\sigma} = \dfrac{24.9 - 25}{0.07} \approx -1.43$

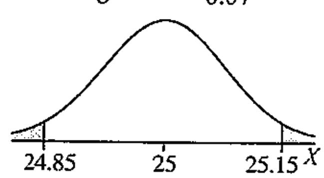


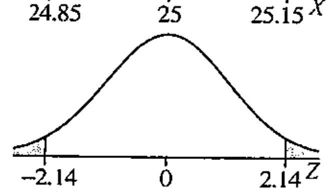


From Table V, the area to the left of $z = -1.43$ is 0.0764, so $P(X < 24.9) =$ 0.0764. [Tech: 0.0766] So, the proportion of rods that has a length less than 24.9 cm is 0.0764, or 7.64%.

(b) $z_1 = \dfrac{x_1 - \mu}{\sigma} = \dfrac{24.85 - 25}{0.07} \approx -2.14$;

$z_2 = \dfrac{x_2 - \mu}{\sigma} = \dfrac{25.15 - 25}{0.07} \approx 2.14$



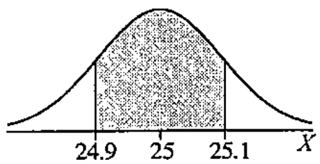


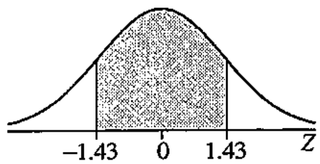
From Table V, the area to the left of $z_1 = -2.14$ is 0.0162, so $P(X < 24.85) =$ 0.0162. The area to the left of $z_2 = 2.14$ is 0.9838, so $P(X > 25.15) = 1 - 0.9838 =$ 0.0162. So, $P(X < 24.85 \text{ or } X > 25.15) =$ $2(0.0162) = 0.0324$. [Tech: 0.0321] The proportion of rods that will be discarded is 0.0324, or 3.24%.

(c) The manager should expect to discard $5000(0.0324) = 162$ of the 5000 steel rods.

(d) $z_1 = \dfrac{x_1 - \mu}{\sigma} = \dfrac{24.9 - 25}{0.07} \approx -1.43$;

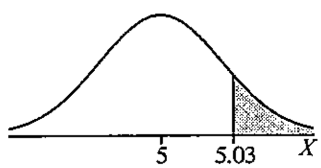
$z_2 = \dfrac{x_2 - \mu}{\sigma} = \dfrac{25.1 - 25}{0.07} \approx 1.43$

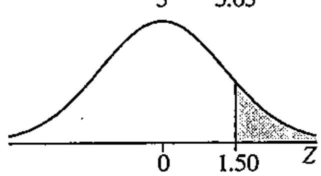




From Table V, the area to the left of $z_1 = -1.43$ is 0.0764 and the area to the left of $z_2 = 1.43$ is 0.9236, so $P(24.9 \le X \le 25.1) = 0.9236 - 0.0764 = 0.8472$. [Tech: 0.8469] So, 0.8472, or 84.72%, of the rods manufactured will be between 24.9 and 25.1 cm. Let n represent the number of rods that must be manufactured. Then, $0.8472n = 10{,}000$, so $n = \dfrac{10{,}000}{0.8472} \approx 11{,}803.59$. Increase this to the next whole number: 11,804. To meet the order, the manager should manufacture 11,804 rods. [Tech: 11,808 rods]

24. (a) $z = \dfrac{x - \mu}{\sigma} = \dfrac{5.03 - 5}{0.02} = 1.5$

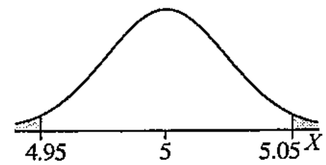


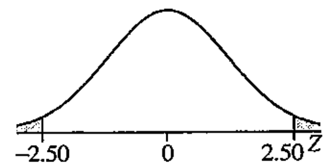


From Table V, the area to the left of $z = 1.50$ is 0.9332, so $P(X > 5.03) = 1 - 0.9332 = 0.0668$. So, the proportion of ball bearings that has a diameter more than 5.03 mm is 0.0668, or 6.68%.

(b) $z_1 = \dfrac{x_1 - \mu}{\sigma} = \dfrac{4.95 - 5}{0.02} = -2.5$;

$z_2 = \dfrac{x_2 - \mu}{\sigma} = \dfrac{5.05 - 5}{0.02} = 2.5$



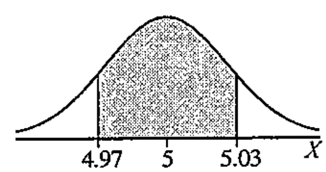


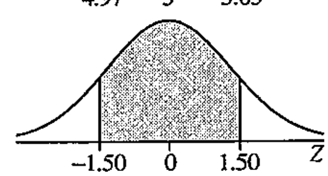
From Table V, the area to the left of $z_1 = -2.50$ is 0.0062, so $P(X < 4.95) = 0.0062$. The area to the left of $z_2 = 2.50$ is 0.9938, so $P(X > 5.05) = 1 - 0.9938 = 0.0062$. So, $P(X < 4.95 \text{ or } X > 5.05) = 2(0.0062) = 0.0124$. The proportion of ball bearings that will be discarded is 0.0124, or 1.24%.

(c) The manager should expect to discard $30{,}000(0.0124) = 372$ of the 30,000 ball bearings.

(d) $z_1 = \dfrac{x_1 - \mu}{\sigma} = \dfrac{4.97 - 5}{0.02} \approx -1.5$;

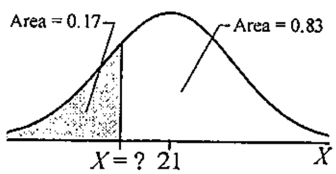
$z_2 = \dfrac{x_2 - \mu}{\sigma} = \dfrac{5.03 - 5}{0.02} \approx 1.5$





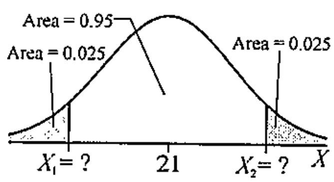
From Table V, the area to the left of $z_1 = -1.50$ is 0.0668 and the area to the left of $z_2 = 1.50$ is 0.9332, so $P(4.97 \le X \le 5.03) = 0.9332 - 0.0668 = 0.8664$. So, 0.8664, or 86.64%, of the ball bearings manufactured will be between 4.97 and 5.03 mm. Let n represent the number of ball bearings that must be manufactured. Then $0.8664n = 50{,}000$, so $n = \dfrac{50{,}000}{0.8664} \approx 57{,}710.06$. Increase this to the next whole number: 57,711. To meet the order, the plant manager should manufacture 57,711 ball bearings.

25. (a) The figure below shows the normal curve with the unknown value of X separating the bottom 17% of the distribution from the top 83%.



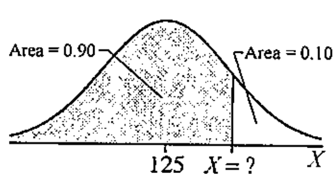
From Table V, the area closest to 0.17 is 0.1711, which corresponds to the z-score -0.95. So, the 17th percentile for incubation times of fertilized chicken eggs is $x = \mu + z\sigma = 21 + (-0.95)(1) \approx 20$ days.

(b) The figure below shows the normal curve with the unknown values of X separating the middle 95% of the distribution from the bottom 2.5% and the top 2.5%.



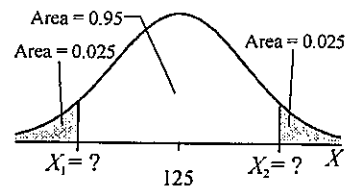
From Table V, the area 0.0250 corresponds to the z-score -1.96. Likewise, the area $0.0250 + .095 = 0.975$ corresponds to the z-score 1.96. Now, $x_1 = \mu + z_1\sigma = 21 + (-1.96)(1) \approx 19$ and $x_2 = \mu + z_2\sigma = 21 + 1.96(1) \approx 23$. Thus, the incubation times that make up the middle 95% of fertilized chicken eggs is between 19 and 23 days.

26. (a) The figure below shows the normal curve with the unknown value of X separating the bottom 90% of the distribution from the top 10%.



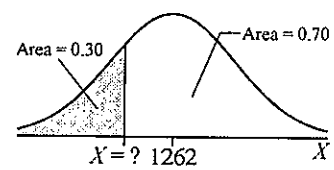
From Table V, the area closest to 0.90 is 0.8997, which corresponds to the z-score 1.28. So, the 90th percentile for the reading speed of sixth-grade students is $x = \mu + z\sigma = 125 + 1.28(24) \approx 156$ words per minute.

(b) The figure that follows shows the normal curve with the unknown values of X separating the middle 95% of the distribution from the bottom 2.5% and the top 2.5%.



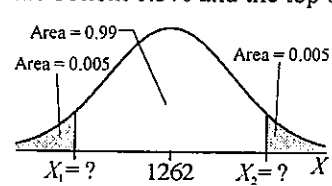
From Table V, the area 0.0250 corresponds to the z-score -1.96. Likewise, the area $0.0250 + .095 = 0.975$ corresponds to the z-score 1.96. Now, $x_1 = \mu + z_1\sigma = 125 + (-1.96)(24) \approx 78$ and $x_2 = \mu + z_2\sigma = 125 + 1.96(24) \approx 172$. So, the cutoffs for unusual reading times are 78 and 172 words per minute.

27. (a) The figure below shows the normal curve with the unknown value of X separating the bottom 30% of the distribution from the top 70%.



From Table V, the area closest to 0.30 is 0.3015, which corresponds to the z-score -0.52. So, the 30th percentile for the number of chocolate chips in an 18-ounce bag of Chips Ahoy! cookies is $x = \mu + z\sigma = 1262 + (-0.52)(118) \approx 1201$ chocolate chips. [Tech: 1200 chocolate chips]

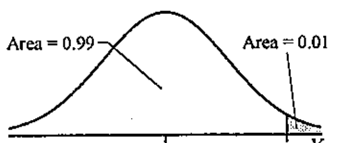
(b) The figure below shows the normal curve with the unknown values of X separating the middle 99% of the distribution from the bottom 0.5% and the top 0.5%.



From Table V, the areas 0.0049 and 0.0051 are equally close to 0.005. We average the corresponding z-scores -2.58 and -2.57 to obtain $z_1 = -2.575$. Likewise, the area

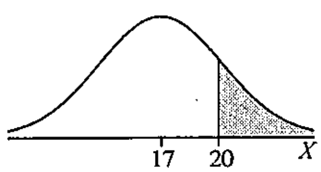
$0.005 + 0.99 = 0.995$ is equally close 0. 9949 and 0.9951. We average the corresponding z-scores 2.57 and 2.58 to obtain $z_2 = 2.575$. Now, $x_1 = \mu + z_1\sigma = 1262 + (-2.575)(118) \approx 958$ and $x_2 = \mu + z_2\sigma = 1262 + 2.575(118) \approx 1566$. So, the number of chocolate chips that make up the middle 99% of 18-ounce bags of Chips Ahoy! cookies is 958 to 1566 chips.

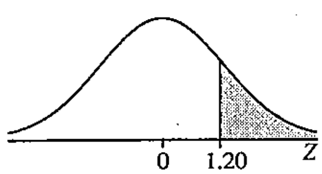
28. The figure below shows the normal curve with the unknown value of X separating the top 1% of the distribution from the bottom 99%.



From Table V, the area closest to 0.99 is 0.9901, which corresponds to the z-score 2.33. So, $x = \mu + z\sigma = 138.5 + 2.33(29) \approx 206$. Wendy's should offer a free meal to any customer who must wait more than 206 seconds.

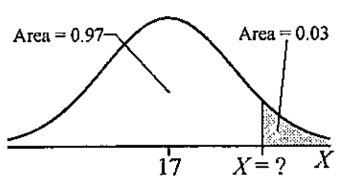
29. **(a)** $z = \dfrac{x-\mu}{\sigma} = \dfrac{20-17}{2.5} = 1.2$





From Table V, the area to the left of $z = 1.20$ is 0.8849, so $P(X > 20) = 1 - 0.8849 = 0.1151$. So, about 11.51% of Speedy Lube's customers receive the service for half price.

(b) The figure below shows the normal curve with the unknown value of X separating the top 3% of the distribution from the bottom 97%.

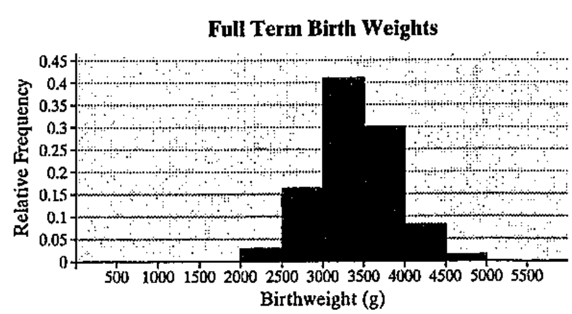


From Table V, the area closest to 0.97 is 0.9699, which corresponds to the z-score 1.88. So, $x = \mu + z\sigma = 17 + 1.88(2.5) \approx 22$. In order to discount only about 3% of its customers, Speedy Lube should make the guaranteed time limit 22 minutes.

30. **(a)** $\Sigma x_i = 22 + 201 + 1{,}1645 + \ldots + 3{,}994$
$= 3{,}344{,}549$

Birth Weight (g)	Relative Frequency
0 – 499	$\dfrac{22}{3{,}344{,}549} < 0.0001$
500 – 999	$\dfrac{201}{3{,}344{,}549} \approx 0.0001$
1,000 – 1,499	$\dfrac{1{,}645}{3{,}344{,}549} \approx 0.0005$
1,500 – 1,999	$\dfrac{9{,}365}{3{,}344{,}549} \approx 0.0028$
2,000 – 2,499	$\dfrac{92{,}191}{3{,}344{,}549} \approx 0.0276$
2,500 – 2,999	$\dfrac{569{,}319}{3{,}344{,}549} \approx 0.1702$
3,000 – 3,499	$\dfrac{1{,}387{,}335}{3{,}344{,}549} \approx 0.4148$
3,500 – 3,999	$\dfrac{988{,}011}{3{,}344{,}549} \approx 0.2954$
4,000 – 4,499	$\dfrac{255{,}700}{3{,}344{,}549} \approx 0.0765$
4,500 – 4,999	$\dfrac{36{,}766}{3{,}344{,}549} \approx 0.0110$
5,000 – 5,499	$\dfrac{3{,}994}{3{,}344{,}549} \approx 0.0012$

(b) The relative frequency histogram is shown below. The distribution is fairly symmetric and bell shaped.



(c) To find the mean, we use the formula $\bar{x} = \dfrac{\Sigma x_i f_i}{\Sigma f_i}$. To find the standard deviation, we use the computational formula $s = \sqrt{s^2} = \sqrt{\dfrac{\Sigma x_i^2 f_i - \dfrac{(\Sigma x_i f_i)^2}{\Sigma f_i}}{(\Sigma f_i) - 1}}$. We organize our computations of x_i, Σf_i, $\Sigma x_i f_i$, and $\Sigma x_i^2 f_i$ in the table that follows:

Class	Midpoint, x_i	Frequency, f_i	$x_i f_i$	x_i^2	$x_i^2 f_i$
0 – 499	$\frac{0+500}{2} = 250$	22	5,500	62,500	1,375,000
500 – 999	$\frac{500+1,000}{2} = 750$	201	150,750	562,500	113,062,500
1,000 – 1,499	1,250	1,645	2,056,250	1,562,500	2,570,312,500
1,500 – 1,999	1,750	9,365	16,388,750	3,062,500	28,680,312,500
2,000 – 2,499	2,250	92,191	207,429,750	5,062,500	466,716,937,500
2,500 – 2,999	2,750	569,319	1,565,627,250	7,562,500	4,305,474,937,500
3,000 – 3,499	3,250	1,387,335	4,508,838,750	10,562,500	14,653,725,937,500
3,500 – 3,999	3,750	988,011	3,705,041,250	14,062,500	13,893,904,687,500
4,000 – 4,499	4,250	255,700	1,086,725,000	18,062,500	4,618,581,250,000
4,500 – 4,999	4,750	36,766	174,638,500	22,562,500	829,532,875,000
5,000 – 5,499	5,250	3,994	20,968,500	27,562,500	110,084,625,000
		$\Sigma f_i =$ 3,344,549	$\Sigma x_i f_i =$ 11,287,870,250		$\Sigma x_i^2 f_i =$ 38,909,386,310,000

With the table complete, we compute the population mean and population standard deviation:

$$\mu = \frac{\Sigma x_i f_i}{\Sigma f_i} = \frac{11,287,870,250}{3,344,549} \approx 3,375 \text{ grams}$$

$$\sigma = \sqrt{\sigma^2} = \sqrt{\frac{\Sigma x_i^2 f_i - \frac{(\Sigma x_i f_i)^2}{\Sigma f_i}}{\Sigma f_i}} = \sqrt{\frac{38,909,386,310,000 - \frac{(11,287,870,250)^2}{3,344,549}}{3,344,549}} \approx 493 \text{ grams}$$

(d) We use technology with $\mu = 3,375$ grams and $\sigma = 493$ grams:

$P(0 \le X \le 499) < 0.0001$

$P(500 \le X \le 999) < 0.0001$

$P(1000 \le X \le 1499) = 0.0001$

$P(1500 \le X \le 1999) = 0.0026$

$P(2000 \le X \le 2499) = 0.0352$

$P(2500 \le X \le 2999) = 0.1849$

$P(3000 \le X \le 3499) = 0.3759$

$P(3500 \le X \le 3999) = 0.2971$

$P(4000 \le X \le 4499) = 0.0911$

$P(4500 \le X \le 4999) = 0.0108$

$P(5000 \le X \le 5499) = 0.0005$

(e) Though the results from the normal model do not perfectly match the actual probabilities, they are generally close in proximity. Therefore, the normal model appears to be effective in describing the birth weights of babies

Consumer Reports®: Sunscreens

(a) There tends to be a great deal of variation in the way people respond to treatments such as sunscreen. Hence, we can either test sunscreens on a very large sample of people or blocking on people, i.e., measuring each person before treatment and then again after treatment. By taking before and after measurements for each person we can determine if there is a treatment effect without testing a very large number of people.

(b) The random assignment of people and application sites to each treatment (the sunscreen) is important to avoid any potential sources of bias.

(c) Product A: $P(X<15)=P\left(Z<\frac{15-15.5}{1.5}\right)$
$\approx P(Z<-0.33)$
$=0.3707$ [Tech: 0.3694]

Product B: $P(X<15)=P\left(Z<\frac{15-14.7}{1.2}\right)$
$=P(Z<0.25)$
$=0.5987$

(d) Product A:
$P(X>17.5)=P\left(Z>\frac{17.5-15.5}{1.5}\right)$
$=P(Z>1.33)$
$=0.0918$ [Tech: 0.0912]

Product B:
$P(X>17.5)=P\left(Z>\frac{17.5-14.7}{1.2}\right)$
$=P(Z>2.33)$
$=0.0099$ [Tech: 0.0098]

(e) Product A:
$P(14.5<X<15.5)$
$=P\left(\frac{14.5-15.5}{1.5}<Z<\frac{15.5-15.5}{1.5}\right)$
$=P(-0.67<Z<0)$
$=0.5-0.2514=0.2486$ [Tech: 0.2475]

Product B:
$P(14.5<X<15.5)$
$=P\left(\frac{14.5-14.7}{1.2}<Z<\frac{15.5-14.7}{1.2}\right)$
$=P(-0.17<Z<0.67)$
$=0.7486-0.4325=0.3161$ [Tech: 0.3137]

(f) Although Product B has a smaller standard deviation, it also has a smaller mean. Since it is important for a product to meet its advertised claim, it appears as if Product A is slightly superior to Product B. Note that approximately 37% of Product A fails to meet its claim while approximately 60% (or more than half) of Product B fails to meet its claim.

Section 7.4

1. Explanations will vary. One possibility follows: Normal random variables are linearly related to their z-scores (by the formula $X=\mu+Z\sigma$), so the plot of values of X against their expected z-scores should be linear.

2. f_i represents the expected proportion of observations less than or equal to the ith observation.

3. The plotted points do not lie within the provided bounds, so the sample data do not come from a normally distributed population.

4. The normal probability plot is roughly linear and all the data lie within the provided bounds, so the sample data could come from a normally distributed population.

5. The plotted points do not lie within the provided bounds, so the sample data do not come from a normally distributed population.

6. The plotted points do not lie within the provided bounds, so the sample data do not come from a normally distributed population.

7. The normal probability plot is roughly linear and all the data lie within the provided bounds, so the sample data could come from a normally distributed population.

8. The normal probability plot is roughly linear and all the data lie within the provided bounds, so the sample data could come from a normally distributed population.

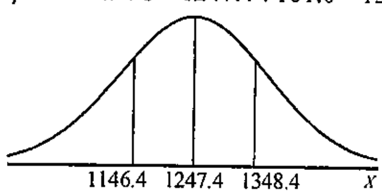
9. **(a)** The normal probability plot is roughly linear, and all the data lie within the provided bounds, so the sample data could come from a population that is normally distributed.

(b) $\Sigma x = 48{,}895$; $\Sigma x^2 = 61{,}211{,}861$; $n = 40$

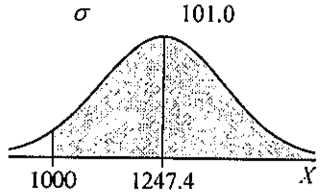
$$\bar{x} = \frac{\Sigma x}{n} = \frac{49{,}895}{40} \approx 1247.4 \text{ chips};$$

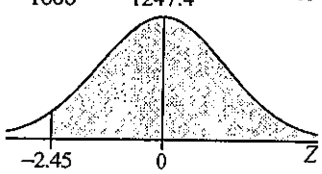
$$s = \sqrt{\frac{\Sigma x^2 - \frac{(\Sigma x)^2}{n}}{n-1}} = \sqrt{\frac{62{,}635{,}301 - \frac{(49{,}895)^2}{40}}{40-1}} \approx 101.0 \text{ chips}$$

(c) $\mu - \sigma \approx \bar{x} - s = 1247.4 - 101.0 = 1146.4$;
$\mu + \sigma \approx \bar{x} + s = 1247.4 + 101.0 = 1348.4$



(d) $z = \frac{x - \mu}{\sigma} = \frac{1000 - 1247.4}{101.0} \approx -2.45$

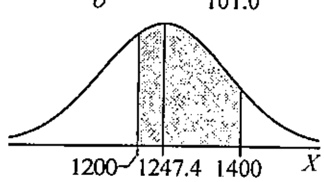


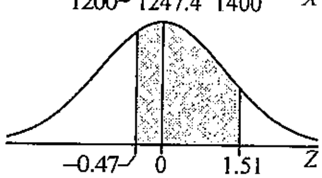


From Table V, the area to the left of $z = -2.45$ is 0.0071, so $P(X > 1000) = 1 - 0.0071 = 0.9929$. [Tech: 0.9928]

(e) $z_1 = \frac{x_1 - \mu}{\sigma} = \frac{1200 - 1247.4}{101.0} \approx -0.47$;

$z_2 = \frac{x_2 - \mu}{\sigma} = \frac{1400 - 1247.4}{101.0} \approx 1.51$





From Table V, the area to the left of $z_1 = -0.47$ is 0.3192 and the area to the left of $z_2 = 1.51$ is 0.9345. So, $P(1200 \le X \le 1400) = 0.9345 - 0.3192 = 0.6153$. [Tech: 0.6152] The proportion of 18-ounce bags of Chips Ahoy! that contains between 1200 and 1400 chips is 0.6153, or 61.53%.

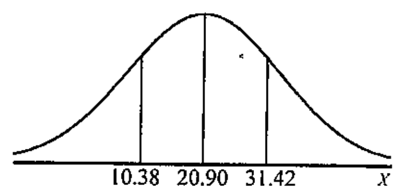
10. (a) The normal probability plot is roughly linear, and all the data lie within the provided bounds, so the sample data could come from a population that is normally distributed.

(b) $\Sigma x = 522.6$; $\Sigma x^2 = 13{,}578.84$; $n = 25$

$$\bar{x} = \frac{\Sigma x}{n} = \frac{522.6}{25} \approx 20.90 \text{ hours};$$

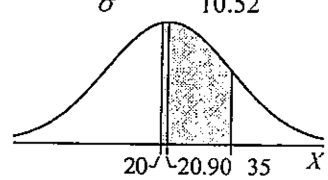
$$s = \sqrt{\frac{\Sigma x^2 - \frac{(\Sigma x)^2}{n}}{n-1}} = \sqrt{\frac{13{,}578.84 - \frac{(522.6)^2}{25}}{25-1}} \approx 10.52 \text{ hours}$$

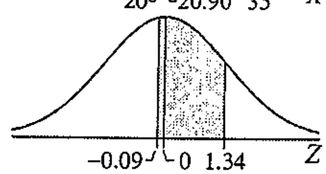
(c) $\mu - \sigma \approx \bar{x} - s = 20.90 - 10.52 = 10.38$;
$\mu + \sigma \approx \bar{x} + s = 20.90 + 10.52 = 31.42$.



(d) $z_1 = \frac{x_1 - \mu}{\sigma} = \frac{20 - 20.90}{10.52} \approx -0.09$;

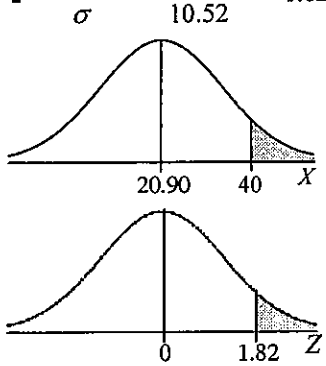
$z_2 = \frac{x_2 - \mu}{\sigma} = \frac{35 - 20.90}{10.52} \approx 1.34$





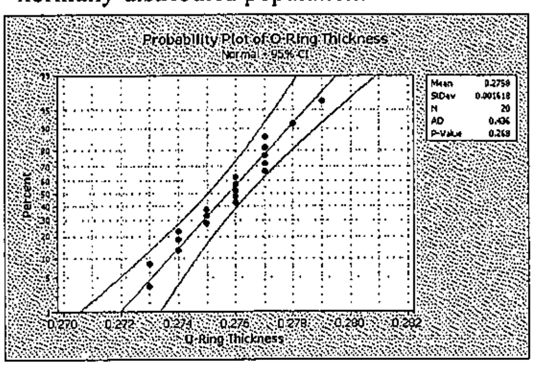
From Table V, the area to the left of $z_1 = -0.09$ is 0.4641 and the area to the left of $z_2 = 1.34$ is 0.9099. So, $P(20 \le X \le 35) = 0.9099 - 0.4641 = 0.4458$. [Tech: 0.4440]

(e) $z = \dfrac{x-\mu}{\sigma} = \dfrac{40-20.90}{10.52} \approx 1.82$

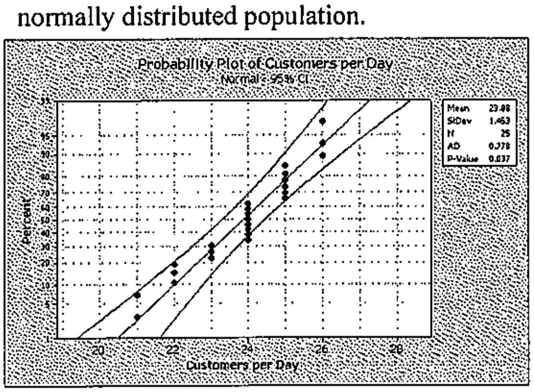


From Table V, the area to the left of $z = 1.82$ is 0.9656, so $P(X > 40) = 1 - 0.9656 = 0.0344$. Thus, the proportion of college students aged 18 to 24 years who watch more than 40 hours of television per week is 0.0344, or 3.44%.

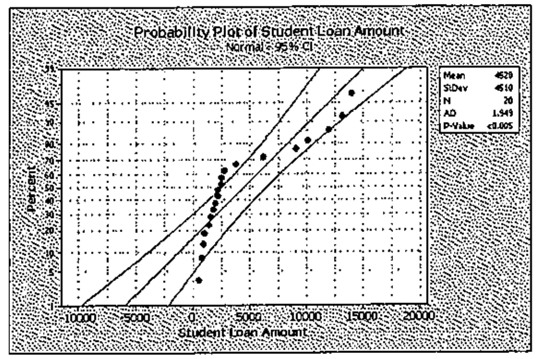
11. The normal probability plot is approximately linear, so the sample data could come from a normally distributed population.



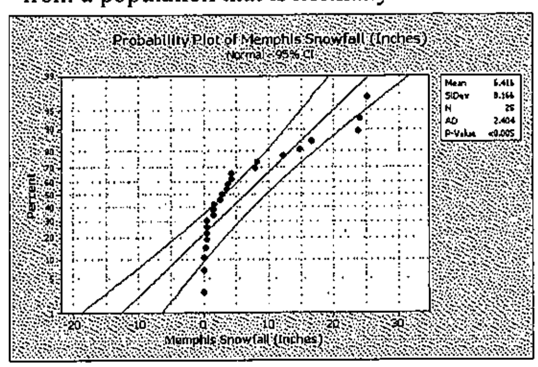
12. The normal probability plot is approximately linear, so the sample data could come from a normally distributed population.



13. The normal probability plot is not approximately linear (points lie outside the provided bounds), so the sample data do not come from a normally distributed population.



14. The plotted points are not linear and do not lie within the provided bounds, so the data are not from a population that is normally distributed.



Section 7.5

1. A probability experiment is a binomial experiment if all the following are true:
   (1) The experiment is performed n independent times.
   (2) For each trial, there are two mutually exclusive outcomes – success or failure.
   (3) The probability of success, p, is the same for each trial of the experiment.

2. The normal distribution can used to approximate binomial probabilities if $np(1-p) \geq 10$.

3. We must use a correction for continuity when using the normal distribution to approximate binomial probabilities because we are using a continuous density function to approximate the probability of a discrete random variable.

4. False. If X is a binomial random variable, then $P(3 \leq X < 7) = P(3 \leq X \leq 6)$. Therefore, to approximate the probability using a normal probability distribution, we compute $P(3-0.5 \leq X \leq 6+0.5) = P(2.5 \leq X \leq 6.5)$.

5. Approximate $P(X \geq 40)$ by computing the area under the normal curve to the right of $x = 39.5$.

6. Approximate $P(X \leq 20)$ by computing the area under the normal curve to the left of $x = 20.5$.

7. Approximate $P(X = 8)$ by computing the area under the normal curve between $x = 7.5$ and $x = 8.5$.

8. Approximate $P(X = 12)$ by computing the area under the normal curve between $x = 11.5$ and $x = 12.5$.

9. Approximate $P(18 \leq X \leq 24)$ by computing the area under the normal curve between $x = 17.5$ and $x = 24.5$.

10. Approximate $P(30 \leq X \leq 40)$ by computing the area under the normal curve between $x = 29.5$ and $x = 40.5$.

11. Approximate $P(X > 20) = P(X \geq 21)$ by computing the area under the normal curve to the right of $x = 20.5$.

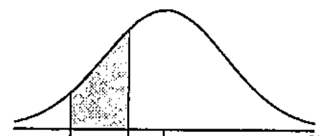
12. Approximate $P(X < 40) = P(X \leq 39)$ by computing the area under the normal curve to the left of $x = 39.5$.

13. Approximate $P(X > 500) = P(X \geq 501)$ by computing the area under the normal curve to the right of $x = 500.5$.

14. Approximate $P(X < 35) = P(X \leq 34)$ by computing the area under the normal curve to the left of $x = 34.5$.

15. Using $P(x) = {}_nC_x p^x (1-p)^{n-x}$, with the parameters $n = 60$ and $p = 0.4$, we get $P(20) = {}_{60}C_{20}(0.4)^{20}(0.6)^{40} \approx 0.0616$. Now $np(1-p) = 60 \cdot 0.4 \cdot (1-0.4) = 14.4 \geq 10$, so the normal approximation can be used, with $\mu_X = np = 60(0.4) = 24$ and $\sigma_X = \sqrt{np(1-p)}$ $= \sqrt{14.4} \approx 3.795$. With continuity correction we calculate:

$$
\begin{aligned}
P(20) &\approx P(19.5 < X < 20.5) \\
&= P\left(\frac{19.5-24}{\sqrt{14.4}} < Z < \frac{20.5-24}{\sqrt{14.4}}\right) \\
&= P(-1.19 < Z < -0.92) \\
&= 0.1788 - 0.1170 \\
&= 0.0618 \quad \text{[Tech: 0.0603]}
\end{aligned}
$$



16. Using $P(x) = {}_nC_x p^x (1-p)^{n-x}$, with the parameters $n = 80$ and $p = 0.15$, we get $P(18) = {}_{80}C_{18}(0.15)^{18}(0.85)^{62} \approx 0.0221$. Now $np(1-p) = 80 \cdot 0.15 \cdot (1-0.15) = 10.2 \geq 10$, so the normal approximation can be used, with $\mu_X = np = 80(0.15) = 12$ and $\sigma_X = \sqrt{np(1-p)}$ $= \sqrt{10.2} \approx 3.194$. With continuity correction we calculate:

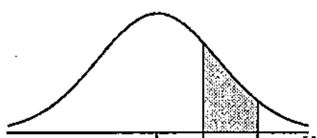
$$
\begin{aligned}
P(18) &\approx P(17.5 < X < 18.5) \\
&= P\left(\frac{17.5-12}{\sqrt{10.2}} < Z < \frac{18.5-12}{\sqrt{10.2}}\right) \\
&= P(1.72 < Z < 2.04) \\
&= 0.9793 - 0.9573 \\
&= 0.0220 \quad \text{[Tech: 0.0216]}
\end{aligned}
$$

17. Using $P(x) = {}_nC_x p^x (1-p)^{n-x}$, with the parameters $n = 40$ and $p = 0.25$, we get $P(30) = {}_{40}C_{30}(0.25)^{30}(0.75)^{70} \approx 4.1 \times 10^{-11}$. Now $np(1-p) = 40 \cdot 0.25 \cdot (1-0.25) = 7.5$, which is below 10, so the normal approximation cannot be used.

18. Using $P(x) = {}_nC_x p^x (1-p)^{n-x}$, with the parameters $n = 100$ and $p = 0.05$, we get $P(50) = {}_{100}C_{50}(0.05)^{50}(0.95)^{50} \approx 6.9 \times 10^{-38}$. Now $np(1-p) = 100 \cdot 0.05 \cdot (1-0.05) = 4.75$, which is below 10, so the normal approximation cannot be used.

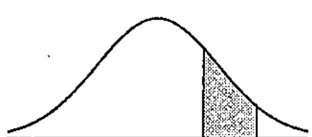
19. Using $P(x) = {}_nC_x p^x(1-p)^{n-x}$, with the parameters $n = 75$ and $p = 0.75$, we get $P(60) = {}_{75}C_{60}(0.75)^{60}(0.25)^{15} \approx 0.0677$. Now $np(1-p) = 75 \cdot 0.75 \cdot (1-0.75) = 14.0625 \geq 10$, so the normal approximation can be used, with $\mu_X = 75(0.75) = 56.25$ and $\sigma_X = \sqrt{np(1-p)} = \sqrt{14.0625} = 3.75$. With continuity correction we calculate:

$$\begin{aligned} P(60) &\approx P(59.5 < X < 60.5) \\ &= P\left(\frac{59.5-56.25}{3.75} < Z < \frac{60.5-56.25}{3.75}\right) \\ &= P(0.87 < Z < 1.13) \\ &= 0.8707 - 0.8078 \\ &= 0.0630 \quad \text{[Tech: 0.0645]} \end{aligned}$$



20. Using $P(x) = {}_nC_x p^x(1-p)^{n-x}$, with the parameters $n = 85$ and $p = 0.8$, we get $P(70) = {}_{85}C_{70}(0.8)^{70}(0.2)^{15} \approx 0.0970$. Now $np(1-p) = 85 \cdot 0.8 \cdot (1-0.8) = 13.6 \geq 10$, so the normal approximation can be used, with $\mu_X = 85(0.8) = 68$ and $\sigma_X = \sqrt{np(1-p)} = \sqrt{13.6} \approx 3.688$. With continuity correction we calculate:

$$\begin{aligned} P(70) &\approx P(69.5 < X < 70.5) \\ &= P\left(\frac{69.5-68}{\sqrt{13.6}} < Z < \frac{70.5-68}{\sqrt{13.6}}\right) \\ &= P(0.41 < Z < 0.68) \\ &= 0.7517 - 0.6591 \\ &= 0.0926 \quad \text{[Tech: 0.0932]} \end{aligned}$$



21. From the parameters $n = 150$ and $p = 0.9$, we get $\mu_X = np = 150 \cdot 0.9 = 135$ and $\sigma_X = \sqrt{np(1-p)} = \sqrt{150 \cdot 0.9 \cdot (1-0.9)} = \sqrt{13.5} \approx 3.674$. Note that $np(1-p) = 13.5 \geq 10$, so the normal approximation to the binomial distribution can be used.

(a) $$\begin{aligned} P(130) &\approx P(129.5 < X < 130.5) \\ &= P\left(\frac{129.5-135}{\sqrt{13.5}} < Z < \frac{130.5-135}{\sqrt{13.5}}\right) \\ &= P(-1.50 < Z < -1.22) \\ &= 0.1112 - 0.0668 \\ &= 0.0444 \quad \text{[Tech: 0.0431]} \end{aligned}$$

(b) $$\begin{aligned} P(X \geq 130) &\approx P(X \geq 129.5) \\ &= P\left(Z \geq \frac{129.5-135}{\sqrt{13.5}}\right) \\ &= P(Z \geq -1.50) \\ &= 1 - 0.0668 \\ &= 0.9332 \quad \text{[Tech: 0.9328]} \end{aligned}$$

(c) $$\begin{aligned} P(X < 125) &= P(X \leq 124) \\ &\approx P(X \leq 124.5) \\ &= P\left(Z \leq \frac{124.5-135}{\sqrt{13.5}}\right) \\ &= P(Z \leq -2.86) \\ &= 0.0021 \end{aligned}$$

(d) $$\begin{aligned} &P(125 \leq X \leq 135) \\ &\approx P(124.5 \leq X \leq 135.5) \\ &= P\left(\frac{124.5-135}{\sqrt{13.5}} < Z < \frac{135.5-135}{\sqrt{13.5}}\right) \\ &= P(-2.86 < Z < 0.14) \\ &= 0.5557 - 0.0021 \\ &= 0.5536 \quad \text{[Tech: 0.5520]} \end{aligned}$$

22. From the parameters $n = 100$ and $p = 0.8$ we get $\mu_X = np = 100 \cdot 0.8 = 80$ and $\sigma_X = \sqrt{np(1-p)} = \sqrt{100 \cdot 0.8 \cdot (1-0.8)} = \sqrt{16} = 4$. Note that $np(1-p) = 16 \geq 10$, so the normal approximation to the binomial distribution can be used.

(a) $$\begin{aligned} P(80) &\approx P(79.5 \leq X \leq 80.5) \\ &= P\left(\frac{79.5-80}{4} \leq Z \leq \frac{80.5-80}{4}\right) \\ &= P(-0.13 \leq Z \leq 0.13) \\ &= 0.5517 - 0.4483 \\ &= 0.1034 \quad \text{[Tech:0.0995]} \end{aligned}$$

(b) $$\begin{aligned} P(X \geq 80) &\approx P(X \geq 79.5) \\ &= P\left(Z \geq \frac{79.5-80}{4}\right) \\ &= P(Z \geq -0.13) \\ &= 1 - 0.4483 \\ &= 0.5517 \quad \text{[Tech: 0.5497]} \end{aligned}$$

(c) $P(X<70)=P(X\le 69)$
$\approx P(X\le 69.5)$
$=P\left(Z\le\frac{69.5-80}{4}\right)$
$=P(Z\le -2.63)$
$=0.0043$

(d) $P(70\le X\le 90)$
$\approx P(69.5\le X\le 90.5)$
$=P\left(\frac{69.5-80}{4}\le Z\le\frac{90.5-80}{4}\right)$
$=P(-2.63\le Z\le 2.63)$
$=0.9957-0.0043$
$=0.9914$ [Tech: 0.9913]

23. From the parameters $n=600$ and $p=0.02$ we get $\mu_X=np=600\cdot 0.02=12$ and $\sigma_X=\sqrt{np(1-p)}=\sqrt{600\cdot 0.02\cdot(1-0.02)}=\sqrt{11.76}\approx 3.429$. Note that $np(1-p)=11.76\ge 10$, so the normal approximation to the binomial distribution can be used.

(a) $P(20)\approx P(19.5\le X\le 20.5)$
$=P\left(\frac{19.5-12}{\sqrt{11.76}}\le Z\le\frac{20.5-12}{\sqrt{11.76}}\right)$
$=P(2.19\le Z\le 2.48)$
$=0.9934-0.9857$
$=0.0077$ [Tech: 0.0078]

(b) $P(X\le 20)\approx P(X\le 20.5)$
$=P\left(Z\le\frac{20.5-12}{\sqrt{11.76}}\right)$
$=P(Z\le 2.48)$
$=0.9934$

(c) $P(X\ge 22)\approx P(X\ge 21.5)$
$=P\left(Z\ge\frac{21.5-12}{\sqrt{11.76}}\right)$
$=P(Z\ge 2.77)$
$=1-0.9972$
$=0.0028$

(d) $P(20\le X\le 30)$
$\approx P(19.5\le X\le 30.5)$
$=P\left(\frac{19.5-12}{\sqrt{11.76}}\le Z\le\frac{30.5-12}{\sqrt{11.76}}\right)$
$=P(2.19\le Z\le 5.39)$
$=1.0000-0.9857$
$=0.0143$ [Tech: 0.0144]

24. From the parameters $n=50$ and $p=0.678$ we get $\mu_X=np=50\cdot 0.678=33.9$ and $\sigma_X=\sqrt{np(1-p)}=\sqrt{50\cdot 0.678\cdot(1-0.678)}=\sqrt{10.9158}\approx 3.304$. Note that $np(1-p)=10.9158\ge 10$, so the normal approximation to the binomial distribution can be used.

(a) $P(40)\approx P(39.5\le X\le 40.5)$
$=P\left(\frac{39.5-33.9}{\sqrt{10.9158}}\le Z\le\frac{40.5-33.9}{\sqrt{10.9158}}\right)$
$=P(1.69\le Z\le 2.00)$
$=0.9772-0.9545$
$=0.0227$ [Tech: 0.0222]

(b) $P(X\ge 35)\approx P(X\ge 34.5)$
$=P\left(Z\ge\frac{34.5-33.9}{\sqrt{10.9158}}\right)$
$=P(Z\ge 0.18)$
$=1-0.5714$
$=0.4286$ [Tech: 0.4279]

(c) $P(X<25)=P(X\le 24)$
$\approx P(X\le 24.5)$
$=P\left(Z\le\frac{24.5-33.9}{\sqrt{10.9158}}\right)$
$=P(Z\le -2.85)$
$=0.0022$

(d) $P(30\le X\le 35)$
$\approx P(29.5\le X\le 35.5)$
$=P\left(\frac{29.5-33.9}{\sqrt{10.9158}}\le Z\le\frac{35.5-33.9}{\sqrt{10.9158}}\right)$
$=P(-1.33\le Z\le 0.48)$
$=0.6844-0.0918$
$=0.5926$ [Tech: 0.5944]

25. From the parameters $n=200$ and $p=0.55$, we get $\mu_X=np=200\cdot 0.55=110$ and $\sigma_X=\sqrt{np(1-p)}=\sqrt{200\cdot 0.55\cdot(1-0.55)}=\sqrt{49.5}\approx 7.036$. Note that $np(1-p)=49.5\ge 10$, so the normal approximation to the binomial distribution can be used.

(a) $P(X\ge 130)\approx P(X\ge 129.5)$
$=P\left(Z\ge\frac{129.5-110}{\sqrt{49.5}}\right)$
$=P(Z\ge 2.77)$
$=1-0.9972$
$=0.0028$

(b) Yes, the result from part (a) contradicts the results of the *Current Population Survey* because the result from part (a) is unusual. Fewer than 3 samples in 1000 will result in 130 or more male students living at home if the true percentage is 55%.

26. From the parameters $n = 200$ and $p = 0.46$, we get $\mu_X = np = 200 \cdot 0.46 = 92$ and $\sigma_X = \sqrt{np(1-p)} = \sqrt{200 \cdot 0.46 \cdot (1-0.46)} = \sqrt{49.68} \approx 7.048$. Note that $np(1-p) = 49.68 \geq 10$, so the normal approximation to the binomial distribution can be used.

(a) $$\begin{aligned} P(X \geq 110) &\approx P(X \geq 109.5) \\ &= P\left(Z \geq \frac{109.5-92}{\sqrt{49.68}}\right) \\ &= P(Z \geq 2.48) \\ &= 1-0.9934 \\ &= 0.0066 \quad \text{[Tech: 0.0065]} \end{aligned}$$

(b) Yes, the result from part (a) contradicts the results of the *Current Population Survey* because the result from part (a) is unusual. Fewer than 7 samples in 1000 will result in 110 or more female students living at home if the true percentage is 46%.

27. From the parameters $n = 150$ and $p = 0.37$, we get $\mu_X = np = 150 \cdot 0.37 = 55.5$ and $\sigma_X = \sqrt{np(1-p)} = \sqrt{150 \cdot 0.37 \cdot (1-0.37)} = \sqrt{34.965} \approx 5.913$. Note that $np(1-p) = 34.965 \geq 10$, so the normal approximation to the binomial distribution can be used.

(a) $$\begin{aligned} P(X \geq 75) &\approx P(X \geq 74.5) \\ &= P\left(Z \geq \frac{74.5-55.5}{\sqrt{34.965}}\right) \\ &= P(Z \geq 3.21) \\ &= 1-0.9993 \\ &= 0.0007 \end{aligned}$$

(b) Yes, the result from part (a) contradicts the results of the *Current Population Survey* because the result from part (a) is unusual. Fewer than 1 sample in 1000 will result in 75 or more respondents preferring a male if the true percentage who prefer a male is 37%.

28. From the parameters $n = 500$ and $p = 0.03$, we get $\mu_X = np = 500 \cdot 0.03 = 15$ and $\sigma_X = \sqrt{np(1-p)} = \sqrt{500 \cdot 0.03 \cdot (1-0.03)} = \sqrt{14.55} \approx 3.814$. Note that $np(1-p) = 14.55 \geq 10$, so the normal approximation to the binomial distribution can be used.

(a) $$\begin{aligned} P(X \geq 20) &\approx P(X \geq 19.5) \\ &= P\left(Z \geq \frac{19.5-15}{\sqrt{14.55}}\right) \\ &= P(Z \geq 1.18) \\ &= 1-0.8810 \\ &= 0.1190 \quad \text{[Tech: 0.1191]} \end{aligned}$$

(b) No, the result form part (a) does not contradict the *USA Today* "Snapshot" because the result from part (a) is not unusual.

Chapter 7 Review Exercises

1. (a) μ is the center (and peak) of the normal distribution, so $\mu = 60$.

(b) σ is the distance from the center to the points of inflection, so $\sigma = 70 - 60 = 10$.

(c) Interpretation 1: The proportion of values of the random variable to the right of $x = 75$ is 0.0668.
Interpretation 2: The probability that a randomly selected value is greater than $x = 75$ is 0.0668.

(d) Interpretation 1: The proportion of values of the random variable between $x = 50$ and $x = 75$ is 0.7745.
Interpretation 2: The probability that a randomly selected value is between $x = 50$ and $x = 75$ is 0.7745.

2. (a) $z_1 = \dfrac{x_1 - \mu}{\sigma} = \dfrac{18-20}{4} = -0.5$

(b) $z_2 = \dfrac{x_2 - \mu}{\sigma} = \dfrac{21-20}{4} = 0.25$

(c) The area between z_1 and z_2 is also 0.2912.