Chapter 8

Sampling Distributions

Section 8.1

1. The sampling distribution of a statistic (such as the sample mean) is a probability distribution for all possible values of the statistic computed from a sample of size n.

2. The Central Limit Theorem states that, regardless of the distribution of the population, the sampling distribution of the sample mean, $\bar{x}$, becomes approximately normal as the sample size, n, increases (provided σ_X is finite).

3. standard error; mean

4. zero

5. The mean of the sampling distribution of $\bar{x}$ is given by $\mu_{\bar{x}} = \mu$ and the standard deviation is given by $\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}}$.

6. To say that the sampling distribution of $\bar{x}$ is normal, the population must be normal. That is, the distribution of X must be normal.

7. four; to see this, note that $\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{4n}} = \dfrac{1}{2} \cdot \dfrac{\sigma}{\sqrt{n}}$.

8. True

9. The sampling distribution of $\bar{x}$ would be exactly normal. The mean and standard deviation would be $\mu_{\bar{x}} = \mu = 30$ and $\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{8}{\sqrt{10}} \approx 2.530$.

10. No, the population does not need to be normally distributed because the sample size is sufficiently large, $n = 40 \geq 30$. The sampling distribution is approximately normal (or exactly normal if the population is normally distributed) with mean $\mu_{\bar{x}} = \mu = 50$ and $\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{4}{\sqrt{40}} = \dfrac{2}{\sqrt{10}} \approx 0.632$.

11. $\mu_{\bar{x}} = \mu = 80$; $\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{14}{\sqrt{49}} = \dfrac{14}{7} = 2$

12. $\mu_{\bar{x}} = \mu = 64$; $\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{18}{\sqrt{36}} = \dfrac{18}{6} = 3$

13. $\mu_{\bar{x}} = \mu = 52$; $\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{10}{\sqrt{21}} \approx 2.182$

14. $\mu_{\bar{x}} = \mu = 27$; $\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{6}{\sqrt{15}} \approx 1.549$

15. **a.** The sampling distribution is symmetric about 500, so the mean is $\mu_{\bar{x}} = 500$.

    b. The inflection points are at 480 and 520, so $\sigma_{\bar{x}} = 520 - 500 = 20$ (or $500 - 480 = 20$).

    c. Since $n = 16 \leq 30$, the population be must be normal so that the sampling distribution of $\bar{x}$ is normal.

    d. $\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}}$

    $20 = \dfrac{\sigma}{\sqrt{16}}$

    $\sigma = 20\sqrt{16} = 20(4) = 80$

    The standard deviation of the population from which the sample is drawn is 80.

16. **a.** The sampling distribution is symmetric about 12, so the mean is $\mu_{\bar{x}} = 12$.

    b. The inflection points are at 11.95 and 12.05, so $\sigma_{\bar{x}} = 12.05 - 12 = 0.05$

    c. Since $n = 9 \leq 30$, the population be must be normal so that the sampling distribution of $\bar{x}$ is normal.

    d. $\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}}$

    $0.05 = \dfrac{\sigma}{\sqrt{9}}$

    $\sigma = 0.05\sqrt{9} = 0.05(3) = 0.15$

    The standard deviation of the population from which the sample is drawn is 0.15.

17. (a) Since $\mu = 80$ and $\sigma = 14$, the mean and standard deviation of the sampling distribution of $\bar{x}$ are given by:

$\mu_{\bar{x}} = \mu = 80$; $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{14}{\sqrt{49}} = \frac{14}{7} = 2$.

We are not told that the population is normally distributed, but we do have a large sample size ($n = 49 \geq 30$). Therefore, we can use the Central Limit Theorem to say that the sampling distribution of $\bar{x}$ is approximately normal.

(b) $$\begin{aligned} P(\bar{x} > 83) &= P\left(Z > \frac{83-80}{2}\right) \\ &= P(Z > 1.50) \\ &= 1 - P(Z \leq 1.50) \\ &= 1 - 0.9332 \\ &= 0.0668 \end{aligned}$$

If we take 100 simple random samples of size $n = 49$ from a population with $\mu = 80$ and $\sigma = 14$, then about 7 of the samples will result in a mean that is greater than 83.

(c) $$\begin{aligned} P(\bar{x} \leq 75.8) &= P\left(Z \leq \frac{75.8-80}{2}\right) \\ &= P(Z \leq -2.10) \\ &= 0.0179 \end{aligned}$$

If we take 100 simple random samples of size $n = 49$ from a population with $\mu = 80$ and $\sigma = 14$, then about 2 of the samples will result in a mean that is less than or equal to 75.8.

(d) $$\begin{aligned} &P(78.3 < \bar{x} < 85.1) \\ &= P\left(\frac{78.3-80}{2} < Z < \frac{85.1-80}{2}\right) \\ &= P(-0.85 < Z < 2.55) \\ &= 0.9946 - 0.1977 \\ &= 0.7969 \quad \text{[Tech: 0.7970]} \end{aligned}$$

If we take 100 simple random samples of size $n = 49$ from a population with $\mu = 80$ and $\sigma = 14$, then about 78 of the samples will result in a mean that is between 78.3 and 85.1.

18. (a) Since $\mu = 64$ and $\sigma = 18$, the mean and standard deviation of the sampling distribution of $\bar{x}$ are given by:

$\mu_{\bar{x}} = \mu = 64$; $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{18}{\sqrt{36}} = \frac{18}{6} = 3$

We are not told that the population is normally distributed, but we do have a large sample size ($n = 36 \geq 30$). Therefore, we can use the Central Limit Theorem to say that the sampling distribution of $\bar{x}$ is approximately normal.

(b) $$\begin{aligned} P(\bar{x} < 62.6) &= P\left(Z < \frac{62.6-64}{3}\right) \\ &= P(Z < -0.47) \\ &= 0.3192 \quad \text{[Tech:0.3204]} \end{aligned}$$

If we take 100 simple random samples of size $n = 36$ from a population with $\mu = 64$ and $\sigma = 18$, then about 32 of the samples will result in a mean that is less than 62.6.

(c) $$\begin{aligned} P(\bar{x} \geq 68.7) &= P\left(Z \geq \frac{68.7-64}{3}\right) \\ &= P(Z \geq 1.57) \\ &= 1 - P(Z < 1.57) \\ &= 1 - 0.9418 \\ &= 0.0582 \quad \text{[Tech:0.0586]} \end{aligned}$$

If we take 100 simple random samples of size $n = 36$ from a population with $\mu = 64$ and $\sigma = 18$ then about 6 of the samples will result in a mean that is greater than or equal to 68.7.

(d) $$\begin{aligned} &P(59.8 < \bar{x} < 65.9) \\ &= P\left(\frac{59.8-64}{3} < \bar{x} < \frac{65.9-64}{3}\right) \\ &= P(-1.40 < Z < 0.63) \\ &= 0.7357 - 0.0808 \\ &= 0.6549 \quad \text{[Tech:0.6560]} \end{aligned}$$

If we take 100 simple random samples of size $n = 36$ from a population with $\mu = 64$ and $\sigma = 18$, then about 65 of the samples will result in a mean that is between 59.8 and 65.9.

19. (a) The population must be normally distributed. If this is the case, then the sampling distribution of $\bar{x}$ is exactly normal. The mean and standard deviation of the sampling distribution are $\mu_{\bar{x}} = \mu = 64$ and

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{17}{\sqrt{12}} \approx 4.907 .$$

(b)
$$\begin{aligned} P(\bar{x} < 67.3) &= P\left(Z < \frac{67.3-64}{17/\sqrt{12}}\right) \\ &= P(Z < 0.67) \\ &= 0.7486 \quad \text{[Tech: 0.7493]} \end{aligned}$$

If we take 100 simple random samples of size $n = 12$ from a population that is normally distributed with $\mu = 64$ and $\sigma = 17$, then about 75 of the samples will result in a mean that is less than 67.3.

(c)
$$\begin{aligned} P(\bar{x} \geq 65.2) &= P\left(Z \geq \frac{65.2-64}{17/\sqrt{12}}\right) \\ &= P(Z \geq 0.24) \\ &= 1 - P(Z < 0.24) \\ &= 1 - 0.5948 \\ &= 0.4052 \quad \text{[Tech: 0.4034]} \end{aligned}$$

If we take 100 simple random samples of size $n = 12$ from a population that is normally distributed with $\mu = 64$ and $\sigma = 17$ then about 41 of the samples will result in a mean that is greater than or equal to 65.2.

20. (a) The population must be normally distributed. If this is the case, then the sampling distribution of $\bar{x}$ is exactly normal. The mean and standard deviation of the sampling distribution are $\mu_{\bar{x}} = \mu = 64$ and

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{17}{\sqrt{20}} \approx 3.801 .$$

(b)
$$\begin{aligned} P(\bar{x} < 67.3) &= P\left(Z < \frac{67.3-64}{17/\sqrt{20}}\right) \\ &= P(Z < 0.87) \\ &= 0.8078 \quad \text{[Tech: 0.8073]} \end{aligned}$$

If we take 100 simple random samples of size $n = 20$ from a population that is normally distributed with $\mu = 64$ and $\sigma = 17$, then about 81 of the samples will result in a mean that is less than 67.3.

(c)
$$\begin{aligned} P(\bar{x} \geq 65.2) &= P\left(Z \geq \frac{65.2-64}{17/\sqrt{12}}\right) \\ &= P(Z \geq 0.32) \\ &= 1 - P(Z < 0.32) \\ &= 1 - 0.6255 \\ &= 0.3745 \quad \text{[Tech: 0.3761]} \end{aligned}$$

If we take 100 simple random samples of size $n = 20$ from a population that is normally distributed with $\mu = 64$ and $\sigma = 17$ then about 37 of the samples will result in a mean that is greater than or equal to 65.2.

(d) Answers will vary. The effect depends on the nature of the probability—some are increased and some are decreased. For example, increasing the sample size increases the probability that $\bar{x} < 67.3$ but decreases the probability that $\bar{x} \geq 65.2$. This happens because $\sigma_{\bar{x}}$ decreases as n increases. The standard deviation of the sampling distribution in Problem 20 is smaller than the standard deviation of the sampling distribution in Problem 19.

21. (a)
$$\begin{aligned} P(X < 260) &= P\left(Z < \frac{260-266}{16}\right) \\ &= P(Z < -0.38) \\ &= 0.3520 \quad \text{[Tech: 0.3538]} \end{aligned}$$

If we select a simple random sample of $n = 100$ human pregnancies, then about 35 of the pregnancies would last less than 260 days.

(b) Since the length of human pregnancies is normally distributed, the sampling distribution of $\bar{x}$ is normal with $\mu_{\bar{x}} = 266$ and $\sigma_{\bar{x}} = \frac{16}{\sqrt{20}} \approx 3.578$.

(c)
$$\begin{aligned} P(\bar{x} < 260) &= P\left(Z < \frac{260-266}{16/\sqrt{20}}\right) \\ &= P(Z < -1.68) \\ &= 0.0465 \quad \text{[Tech: 0.0468]} \end{aligned}$$

If we take 100 simple random samples of size $n = 20$ human pregnancies, then about 5 of the samples will result in a mean gestation period of 260 days or less.

(d) $\mu_{\bar{x}} = \mu = 266$; $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{16}{\sqrt{50}}$

$$P(\bar{x} < 260) = P\left(Z < \frac{260-266}{16/\sqrt{50}}\right)$$
$$= P(Z < -2.65)$$
$$= 0.0040$$

If we take 1000 simple random samples of size $n = 50$ human pregnancies, then about 4 of the samples will result in a mean gestation period of 260 days or less.

(e) Answers will vary. Part (d) indicates that this result would be an unusual observation. Therefore, we would conclude that the sample likely came from a population whose mean gestation period is less than 266 days.

(f) $\mu_{\bar{x}} = \mu = 266$; $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{16}{\sqrt{15}}$

$$P(256 \le \bar{x} \le 276)$$
$$= P\left(\frac{256-266}{16/\sqrt{15}} \le Z \le \frac{276-266}{16/\sqrt{15}}\right)$$
$$= P(-2.42 \le Z \le 2.42)$$
$$= 0.9922 - 0.0078$$
$$= 0.9844 \quad \text{[Tech: 0.9845]}$$

If we take 100 simple random samples of size $n = 15$ human pregnancies, then about 98 of the samples will result in a mean gestation period between 256 and 276 days, inclusive.

22. (a) $P(X < 40) = P\left(Z < \frac{40-43.7}{4.2}\right)$

$$= P(Z < -0.88)$$
$$= 0.1894 \quad \text{[Tech: 0.1892]}$$

If we select a simple random sample of $n = 100$ males ages 20 – 29, then about 19 of the males would have an upper leg length that is less than 40 cm.

(b) $\mu_{\bar{x}} = \mu = 43.7$; $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{4.2}{\sqrt{9}} = 1.4$

$$P(\bar{x} < 40) = P\left(Z < \frac{40-43.7}{1.4}\right)$$
$$= P(Z < -2.64)$$
$$= 0.0041$$

If we take 1000 simple random samples of size $n = 9$ males ages 20 – 29, then about 4 of the samples will result in a mean upper leg length that is less than 40 cm.

(c) $\mu_{\bar{x}} = \mu = 43.7$; $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{4.2}{\sqrt{12}}$

$$P(\bar{x} < 40) = P\left(Z < \frac{40-43.7}{4.2/\sqrt{12}}\right)$$
$$= P(Z < -3.05)$$
$$= 0.0011$$

If we take 1000 simple random samples of size $n = 12$ males ages 20 – 29, then about 1 of the samples will result in a mean upper leg length that is less than 40 cm.

(d) Increasing the sample size decreases the probability that $\bar{x} < 40$. This happens because $\sigma_{\bar{x}}$ decreases as n increases.

(e) Yes, this result would be unusual because, if $\mu_{\bar{x}} = \mu = 43.7$ and $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{4.2}{\sqrt{15}}$, then

$$P(\bar{x} \ge 46) = P\left(Z \ge \frac{46-43.7}{4.2/\sqrt{15}}\right)$$
$$= P(Z \ge 2.12)$$
$$= 1 - P(Z < 2.12)$$
$$= 1 - 0.9830$$
$$= 0.0170$$

If we take 100 simple random samples of size $n = 15$ males ages 20 – 29, then about 2 of the samples will result in a mean upper leg length that is 46 cm or greater. This result qualifies as unusual.

23. (a) $P(X > 95) = P\left(Z > \frac{95-90}{10}\right)$

$$= P(Z > 0.5)$$
$$= 1 - P(Z \le 0.5)$$
$$= 1 - 0.6915$$
$$= 0.3085$$

If we select a simple random sample of $n = 100$ second grade students, then about 31 of the students would read more than 95 words per minute.

(b) $\mu_{\bar{x}} = \mu = 90$; $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{10}{\sqrt{12}}$

$$P(\bar{x} > 95) = P\left(Z > \frac{95-90}{10/\sqrt{12}}\right)$$
$$= P(Z > 1.73)$$
$$= 1 - P(Z \le 1.73)$$
$$= 1 - 0.9582$$
$$= 0.0418 \quad \text{[Tech: 0.0416]}$$

If we take 100 simple random samples of size $n = 12$ second grade students, then about 4 of the samples will result in a mean reading rate that is more than 95 words per minute.

(c) $\mu_{\bar{x}} = \mu = 90$; $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{10}{\sqrt{24}}$

$$\begin{aligned} P(\bar{x} > 95) &= P\left(Z > \frac{95-90}{10/\sqrt{24}}\right) \\ &= P(Z > 2.45) \\ &= 1 - P(Z \le 2.45) \\ &= 1 - 0.9929 \\ &= 0.0071 \quad \text{[Tech: 0.0072]} \end{aligned}$$

If we take 1000 simple random samples of size $n = 24$ second grade students, then about 7 of the samples will result in a mean reading rate that is more than 95 words per minute.

(d) Increasing the sample size decreases the probability that $\bar{x} > 95$. This happens because $\sigma_{\bar{x}}$ decreases as n increases.

(e) No, this result would not be unusual because, if $\mu_{\bar{x}} = \mu = 90$ and $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{10}{\sqrt{20}}$, then

$$\begin{aligned} P(\bar{x} > 92.8) &= P\left(Z > \frac{92.8-90}{10/\sqrt{20}}\right) \\ &= P(Z > 1.25) \\ &= 1 - P(Z \le 1.25) \\ &= 1 - 0.8944 \\ &= 0.1056 \quad \text{[Tech: 0.1052]} \end{aligned}$$

If we take 100 simple random samples of size $n = 20$ second grade students, then about 11 of the samples will result in a mean reading rate that is above 92.8 words per minute. This result does not qualify as unusual. This means that the new reading program is not abundantly more effective than the old program.

24. (a)
$$\begin{aligned} P(X > 95) &= P\left(Z > \frac{95-85}{21.25}\right) \\ &= P(Z > 0.47) \\ &= 1 - P(Z \le 0.47) \\ &= 1 - 0.6808 \\ &= 0.3192 \quad \text{[Tech: 0.3190]} \end{aligned}$$

If we select a simple random sample of $n = 100$ time intervals between eruptions, then about 32 of the intervals would be longer than 95 minutes.

(b) $\mu_{\bar{x}} = \mu = 85$; $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{21.25}{\sqrt{20}}$

$$\begin{aligned} P(\bar{x} > 95) &= P\left(Z > \frac{95-85}{21.25/\sqrt{20}}\right) \\ &= P(Z > 2.10) \\ &= 1 - P(Z \le 2.10) \\ &= 1 - 0.9821 \\ &= 0.0179 \quad \text{[Tech: 0.0177]} \end{aligned}$$

If we take 100 simple random samples of size $n = 20$ time intervals between eruptions, then about 2 of the samples will result in a mean longer than 95 minutes.

(c) $\mu_{\bar{x}} = \mu = 85$; $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{21.25}{\sqrt{30}}$

$$\begin{aligned} P(\bar{x} > 95) &= P\left(Z > \frac{95-85}{21.25/\sqrt{30}}\right) \\ &= P(Z > 2.58) \\ &= 1 - P(Z \le 2.58) \\ &= 1 - 0.9951 \\ &= 0.0049 \quad \text{[Tech: 0.0050]} \end{aligned}$$

If we take 1000 simple random samples of size $n = 30$ time intervals between eruptions, then about 5 of the samples will result in a mean longer than 95 minutes.

(d) Increasing the sample size reduces the probability that $\bar{x} > 95$. This happens because $\sigma_{\bar{x}}$ decreases as n increases.

(e) Answers will vary. From part (c) we see that this is an unusual observation. Therefore, we would conclude that the mean time between eruptions of Old Faithful is actually longer than 85 minutes.

25. (a)
$$\begin{aligned} P(X > 0) &= P\left(Z > \frac{0-0.007233}{0.04135}\right) \\ &= P(Z > -0.17) \\ &= 1 - P(Z \le -0.17) \\ &= 1 - 0.4325 \\ &= 0.5675 \quad \text{[Tech: 0.5694]} \end{aligned}$$

If we select a simple random sample of $n = 100$ months, then about 57 of the months would have positive rates of return.

(b) $\mu_{\bar{x}} = \mu = 0.007233$; $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{0.04135}{\sqrt{12}}$

$$\begin{aligned} P(\bar{x} > 0) &= P\left(Z > \frac{0 - 0.007233}{0.04135/\sqrt{12}}\right) \\ &= P(Z > -0.61) \\ &= 1 - P(Z \le -0.61) \\ &= 1 - 0.2709 \\ &= 0.7291 \quad \text{[Tech: 0.7277]} \end{aligned}$$

If we take 100 simple random samples of size $n = 12$ months, then about 73 of the samples will result in a mean monthly rate that is positive.

(c) $\mu_{\bar{x}} = \mu = 0.007233$; $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{0.04135}{\sqrt{24}}$

$$\begin{aligned} P(\bar{x} > 0) &= P\left(Z > \frac{0 - 0.007233}{0.04135/\sqrt{24}}\right) \\ &= P(Z > -0.86) \\ &= 1 - P(Z \le -0.86) \\ &= 1 - 0.1949 \\ &= 0.8051 \quad \text{[Tech: 0.8043]} \end{aligned}$$

If we take 100 simple random samples of size $n = 24$ months, then about 81 of the samples will result in a mean monthly rate that is positive.

(d) $\mu_{\bar{x}} = \mu = 0.007233$; $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{0.04135}{\sqrt{36}}$

$$\begin{aligned} P(\bar{x} > 0) &= P\left(Z > \frac{0 - 0.007233}{0.04135/\sqrt{36}}\right) \\ &= P(Z > -1.05) \\ &= 1 - P(Z \le -1.05) \\ &= 1 - 0.1469 \\ &= 0.8531 \quad \text{[Tech: 0.8530]} \end{aligned}$$

If we take 100 simple random samples of size $n = 36$ months, then about 85 of the samples will result in a mean monthly rate that is positive.

(e) Answers will vary. Based on the results of parts (b)–(d), the likelihood of earning a positive rate of return increases as the investment time horizon increases.

26. (a) $$\begin{aligned} P(X > 34) &= P\left(Z > \frac{34 - 32}{3.5}\right) \\ &= P(Z > 0.57) \\ &= 1 - P(Z \le 0.57) \\ &= 1 - 0.7157 \\ &= 0.2843 \quad \text{[Tech: 0.2839]} \end{aligned}$$

If we select a simple random sample of $n = 100$ Cobalts, then about 28 of the Cobalts will get more than 34 miles per gallon.

(b) $\mu_{\bar{x}} = \mu = 32$; $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{3.5}{\sqrt{10}}$

$$\begin{aligned} P(\bar{x} > 34) &= P\left(Z > \frac{34 - 32}{3.5/\sqrt{10}}\right) \\ &= P(Z > 1.81) \\ &= 1 - P(Z \le 1.81) \\ &= 1 - 0.9649 \\ &= 0.0351 \quad \text{[Tech: 0.0354]} \end{aligned}$$

If we take 100 simple random samples of size $n = 10$ Cobalts, then about 4 of the samples will result in a mean gasoline mileage that exceeds 34 miles per gallon.

(c) $\mu_{\bar{x}} = \mu = 32$; $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{3.5}{\sqrt{20}}$

$$\begin{aligned} P(\bar{x} > 34) &= P\left(Z > \frac{34 - 32}{3.5/\sqrt{20}}\right) \\ &= P(Z > 2.56) \\ &= 1 - P(Z \le 2.56) \\ &= 1 - 0.9948 \\ &= 0.0052 \quad \text{[Tech: 0.0053]} \end{aligned}$$

If we take 1000 simple random samples of size $n = 20$ Cobalts, then about 5 of the samples will result in a mean gasoline mileage that exceeds 34 miles per gallon. This result would be unusual.

27. (a) Without knowing the shape of the distribution, we would need a sample size of at least 30 so we could apply the Central Limit Theorem.

(b) $\mu_{\bar{x}} = \mu = 11.4$; $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{3.2}{\sqrt{40}}$

$$\begin{aligned} P(\bar{x} < 10) &= P\left(Z < \frac{10 - 11.4}{3.2/\sqrt{40}}\right) \\ &= P(Z < -2.77) \\ &= 0.0028 \end{aligned}$$

If we take 1000 simple random samples of size $n = 40$ oil changes, then about 3 of the samples will result in a mean time less than 10 minutes.

28. (a) Because the distribution of time at the window is skewed right, we would need a sample size of at least 30 in order to apply the Central Limit Theorem and compute probabilities regarding the sample mean.

(b) $P(\bar{x} \le 56.8) = P\left(Z \le \frac{56.8 - 59.3}{13.1/\sqrt{40}}\right)$
$= P(Z \le -1.21)$
$= 0.1131$ [Tech: 0.1137]

If we take 100 simple random samples of size $n = 40$ cars, then about 11 of the samples will result in a mean time of 56.8 seconds or less spent at the window. This result is not unusual, so the new system is not abundantly more effective than the old system.

29. (a) The sampling distribution of $\bar{x}$ is approximately normal because the sample is large, $n = 50 \ge 30$. From the Central Limit Theorem, as the sample size increases, the sampling distribution of the mean becomes more normal.

(b) Assuming that we are sampling from a population that is exactly at the Food Defect Action Level, $\mu_{\bar{x}} = \mu = 3$ and

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{\sqrt{3}}{\sqrt{50}} = \sqrt{\frac{3}{50}} \approx 0.245$.

(c) $P(\bar{x} \ge 3.6) = P\left(Z \ge \frac{3.6 - 3}{\sqrt{3}/\sqrt{50}}\right)$
$= P(Z \ge 2.45)$
$= 1 - 0.9929$
$= 0.0071$ [Tech: 0.0072]

If we take 1000 simple random samples of size $n = 50$ ten-gram portions of peanut butter, then about 7 of the samples will result in a mean of at least 3.6 insect fragments. This result is unusual. We might conclude that the sample comes from a population with a mean higher than 3 insect fragments per ten-gram portion.

30. (a) The sampling distribution of $\bar{x}$ is approximately normal because the sample is large, $n = 40 \ge 30$. From the Central Limit Theorem, as the sample size increases, the sampling distribution of the mean becomes more normal.

(b) $\mu_{\bar{x}} = \mu = 20$ and

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{\sqrt{20}}{\sqrt{40}} = \sqrt{\frac{1}{2}} \approx 0.707$

(c) $P(\bar{x} \ge 22.1) = P\left(Z \ge \frac{22.1 - 20}{\sqrt{20}/\sqrt{40}}\right)$
$= P(Z \ge 2.97)$
$= 1 - 0.9985$
$= 0.0015$

If we take 1000 simple random samples of size $n = 40$ one-hour time periods, then about 2 of the samples will result in a mean of at least 22.1 cars arriving at the drive-through. This result is unusual. We might conclude that business has increased and that the rate of arrival is now greater than 20 cars per hour between 12:00 noon and 1:00 P.M.

31. (a) No, the variable "weekly time spent watching television" is not likely normally distributed. It is likely skewed right.

(b) Because the sample is large, $n = 40 \ge 30$, the sampling distribution of $\bar{x}$ is approximately normal with $\mu_{\bar{x}} = \mu = 2.35$ and $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{1.93}{\sqrt{40}} \approx 0.305$.

(c) $P(2 \le \bar{x} \le 3)$
$= P\left(\frac{2 - 2.35}{1.93/\sqrt{40}} \le Z \le \frac{3 - 2.35}{1.93/\sqrt{40}}\right)$
$= P(-1.15 \le Z \le 2.13)$
$= 0.9834 - 0.1251$
$= 0.8583$ [Tech: 0.8577]

If we take 100 simple random samples of size $n = 40$ adult Americans, then about 86 of the samples will result in a mean time between 2 and 3 hours watching television on a weekday.

(d) $\mu_{\bar{x}} = \mu = 2.35$; $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{1.93}{\sqrt{35}}$

$$P(\bar{x} \le 1.89) = P\left(Z \le \frac{1.89 - 2.35}{1.93/\sqrt{35}}\right)$$
$$= P(Z \le -1.41)$$
$$= 0.0793$$

If we take 100 simple random samples of size $n = 35$ adult Americans, then about 8 of the samples will result in a mean time of 1.89 hours or less watching television on a weekday. This result is not unusual, so this evidence is insufficient to conclude that avid Internet users watch less television.

32. (a) No, the variable "ATM withdrawal" is not likely normally distributed. It is likely skewed right.

(b) Because the sample is large, $n = 50 \ge 30$, the sampling distribution of $\bar{x}$ is approximately normal with $\mu_{\bar{x}} = \mu = 60$ and $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{35}{\sqrt{50}} \approx 4.950$.

(c) $P(70 \le \bar{x} \le 75)$

$$= P\left(\frac{70-60}{35/\sqrt{50}} \le Z \le \frac{75-60}{35/\sqrt{50}}\right)$$
$$= P(2.02 \le Z \le 3.03)$$
$$= 0.9988 - 0.9783$$
$$= 0.0205$$

If we take 100 simple random samples of size $n = 50$ ATM withdrawals, then about 2 of the samples will result in a mean withdrawal amount between \$70 and \$75.

33. (a) $\mu = \frac{\sum x}{N} = \frac{232}{6} \approx 38.7$

The population mean age is 38.7 years.

(b) 37, 43; 37, 29; 37, 47; 37, 36; 37, 40; 43, 29; 43, 47; 43, 36; 43, 40; 29, 47; 29, 36; 29, 40; 47, 36; 47, 40; 36, 40

(c) Obtain each sample mean by adding the two ages in a sample and dividing by two.

$\bar{x} = \frac{37+43}{2} = 40$ yr ; $\bar{x} = \frac{37+29}{2} = 33$ yr ;

$\bar{x} = \frac{37+47}{2} = 42$ yr ; $\bar{x} = \frac{37+36}{2} = 36.5$ yr ,

etc.

$\bar{x}$	$P(\bar{x})$	$\bar{x}$	$P(\bar{x})$
32.5	$1/15 \approx 0.0667$	39.5	$1/15 \approx 0.0667$
33	$1/15 \approx 0.0667$	40	$1/15 \approx 0.0667$
34.5	$1/15 \approx 0.0667$	41.5	$2/15 \approx 0.1333$
36	$1/15 \approx 0.0667$	42	$1/15 \approx 0.0667$
36.5	$1/15 \approx 0.0667$	43.5	$1/15 \approx 0.0667$
38	$2/15 \approx 0.1333$	45	$1/15 \approx 0.0667$
38.5	$1/15 \approx 0.0667$		

(d) $\mu_{\bar{x}} = (32.5)\left(\frac{1}{15}\right) + (33)\left(\frac{1}{15}\right) + \ldots + (45)\left(\frac{1}{15}\right)$

≈ 38.7 years

Notice that this is the same value we obtained in part (a) for the population mean.

(e) $P(35.7 \le \bar{x} \le 41.7)$

$$= \frac{1}{15} + \frac{1}{15} + \frac{2}{15} + \frac{1}{15} + \frac{1}{15} + \frac{1}{15} + \frac{2}{15} = \frac{9}{15} = 0.6$$

(f) <u>for part (b):</u>

37, 43, 29; 37, 43, 47; 37, 43, 36;
37, 43, 40; 37, 29, 47; 37, 29, 36;
37, 29, 40; 37, 47, 36; 37, 47, 40;
37, 36, 40; 43, 29, 47; 43, 29, 36;
43, 29, 40; 43, 47, 36; 43, 47, 40;
43, 36, 40; 29, 47, 36; 29, 47, 40;
29, 36, 40; 47, 36, 40

<u>for part (c):</u>

Obtain each sample mean by adding the three ages in a sample and dividing by three.

$\bar{x} = \frac{37+43+29}{3} \approx 36.3$ yr ;

$\bar{x} = \frac{37+43+47}{3} \approx 42.3$ yr ;

$\bar{x} = \frac{37+43+36}{3} = 38.7$ yr ; etc

$\bar{x}$	$P(\bar{x})$	$\bar{x}$	$P(\bar{x})$
34	$1/20 = 0.05$	39.7	$2/20 = 0.1$
35	$1/20 = 0.05$	40	$2/20 = 0.1$
35.3	$1/20 = 0.05$	41	$1/20 = 0.05$
36	$1/20 = 0.05$	41.3	$1/20 = 0.05$
36.3	$1/20 = 0.05$	42	$1/20 = 0.05$
37.3	$2/20 = 0.1$	42.3	$1/20 = 0.05$
37.7	$2/20 = 0.1$	43.3	$1/20 = 0.05$
38.7	$2/20 = 0.1$		

for part (d):

$$\mu_{\bar{x}} = (34)\left(\frac{1}{20}\right) + (35)\left(\frac{1}{20}\right) + \ldots + (43.3)\left(\frac{1}{20}\right)$$
$$\approx 38.7 \text{ years}$$

Notice that this is the same value we obtained previously.

for part (e):

$$P(35.7 \le \bar{x} \le 41.7) = \frac{14}{20} = \frac{7}{10} = 0.7$$

With the larger sample size, the probability of obtaining a sample mean within 3 years of the population mean has increased.

34. (a) $\mu = \dfrac{\sum x}{N} = \dfrac{854}{6} \approx 142.3$

The population mean running time is 142.3 minutes.

(b) 132, 201; 132, 112; 132, 134, 132, 155; 132, 120; 201, 112; 201, 134; 201, 155; 201, 120; 112, 134; 112, 155; 112, 120; 134, 155, 134, 120; 155, 120

(c) Obtain each sample mean by adding the two running times in a sample and dividing by two.

$\bar{x} = \dfrac{132+201}{2} = 166.5$ min ;

$\bar{x} = \dfrac{132+112}{2} = 122$ min ;

$\bar{x} = \dfrac{132+134}{2} = 133$ min ; etc.

$\bar{x}$	$P(\bar{x})$	$\bar{x}$	$P(\bar{x})$
116	$1/15 \approx 0.0667$	143.5	$1/15 \approx 0.0667$
122	$1/15 \approx 0.0667$	144.5	$1/15 \approx 0.0667$
123	$1/15 \approx 0.0667$	156.5	$1/15 \approx 0.0667$
126	$1/15 \approx 0.0667$	160.5	$1/15 \approx 0.0667$
127	$1/15 \approx 0.0667$	166.5	$1/15 \approx 0.0667$
133	$1/15 \approx 0.0667$	167.5	$1/15 \approx 0.0667$
133.5	$1/15 \approx 0.0667$	178	$1/15 \approx 0.0667$
137.5	$1/15 \approx 0.0667$		

(d) $\mu_{\bar{x}} = \left(\dfrac{1}{15}\right)(116+122+123+\ldots 167.5+178)$

≈ 142.3 minutes

Notice that this is the same value we obtained in part (a) for the population mean.

(e) $P(127.3 \le \bar{x} \le 157.3) = \dfrac{6}{15} = 0.4$

(f) For part (b):

132, 201, 112; 132, 201, 134; 132, 201, 155; 132, 201, 120; 132, 112, 134; 132, 112, 155; 132, 112, 120; 132, 134, 155; 132, 134, 120; 132, 155, 120; 201, 112, 134; 201, 112, 155; 201, 112, 120; 201, 134, 155; 201, 134, 120; 201, 155, 120; 112, 134, 155; 112, 134, 120; 112, 155, 120; 134, 155, 120

For part (c):

$\bar{x}$	$P(\bar{x})$	$\bar{x}$	$P(\bar{x})$
121.3	$1/20 = 0.05$	144.3	$1/20 = 0.05$
122	$1/20 = 0.05$	148.3	$1/20 = 0.05$
126	$1/20 = 0.05$	149	$1/20 = 0.05$
128.7	$1/20 = 0.05$	151	$1/20 = 0.05$
129	$1/20 = 0.05$	151.7	$1/20 = 0.05$
133	$1/20 = 0.05$	155.7	$1/20 = 0.05$
133.7	$1/20 = 0.05$	156	$1/20 = 0.05$
135.7	$1/20 = 0.05$	158.7	$1/20 = 0.05$
136.3	$1/20 = 0.05$	162.7	$1/20 = 0.05$
140.3	$1/20 = 0.05$	163.3	$1/20 = 0.05$

For part (d):

$\mu_{\bar{x}} \approx 142.3$ minutes

Notice that this is the same value we obtained previously.

For part (e):

$P(127.3 \le \bar{x} \le 157.3) = \dfrac{14}{20} = 0.7$;

With the larger sample size, the probability of obtaining a sample mean within 15 minutes of the population mean has increased.

35. (a) – (c) Answers will vary.

(d) $\mu_{\bar{x}} = \mu = 100$; $\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{15}{\sqrt{20}} \approx 3.354$

(e) Answers will vary.

(f)
$$\begin{aligned} P(\bar{x} > 108) &= P\left(Z > \frac{108-100}{15/\sqrt{20}}\right) \\ &= P(Z > 2.39) \\ &= 1 - P(Z \le 2.39) \\ &= 1 - 0.9916 \\ &= 0.0084 \quad \text{[Tech: 0.0085]} \end{aligned}$$

(g) Answers will vary.

36. (a) – (c): In each case, the sampling distribution of $\bar{x}$ is approximately normal with mean $\mu_{\bar{x}} = \mu$ and $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$. Using $\mu = 25$ and $\sigma = 8.23$, we get:

For (a): $\mu_{\bar{x}} = 25;\ \sigma_{\bar{x}} = \frac{8.23}{\sqrt{5}} \approx 3.681$

For (b): $\mu_{\bar{x}} = 25;\ \sigma_{\bar{x}} = \frac{8.23}{\sqrt{10}} \approx 2.603$

For (c): $\mu_{\bar{x}} = 25;\ \sigma_{\bar{x}} = \frac{8.23}{\sqrt{30}} \approx 1.503$

(d) Answers will vary. The overall shapes should be similar, though there should be less variability as the sample size increases.

37. (a) – (b): Answers will vary. Using skewed right with $\mu = 3.92$ and $\sigma = 2.79$, we get:

For (a): $\mu_{\bar{x}} = 3.92;\ \sigma_{\bar{x}} = \frac{2.79}{\sqrt{5}} \approx 1.248$

For (b): $\mu_{\bar{x}} = 3.92;\ \sigma_{\bar{x}} = \frac{2.79}{\sqrt{10}} \approx 0.882$

(c) Specific results from the applet will vary. We can use the Central Limit Theorem to say that the sampling distribution is approximately normal since we have a large sample (that is, $n \geq 30$).

$\mu_{\bar{x}} = 3.92;\ \sigma_{\bar{x}} = \frac{2.79}{\sqrt{50}} \approx 0.395$

(d) Answers will vary. As the sample size increases, we should see the sampling distribution become more normally distributed.

38. (a) – (c): Answers will vary. Using $\mu = 25$ and $\sigma = 14.43$, we get:

For (a): $\mu_{\bar{x}} = 25;\ \sigma_{\bar{x}} = \frac{14.43}{\sqrt{2}} \approx 10.204$

For (b): $\mu_{\bar{x}} = 25;\ \sigma_{\bar{x}} = \frac{14.43}{\sqrt{5}} \approx 6.453$

For (c): $\mu_{\bar{x}} = 25;\ \sigma_{\bar{x}} = \frac{14.43}{\sqrt{10}} \approx 4.563$

(d) Answers will vary. As the sample size increases, we should see the sampling distribution approach a normal distribution.

39. (a) Assuming that only one number is selected, the probability distribution will be as follows:

x	$P(x)$
35	$1/38 \approx 0.0263$
−1	$37/38 \approx 0.9737$

(b) $\mu = (35)(0.0263) + (-1)(0.9737) \approx -\$0.05$

$$\sigma = \sqrt{(35-(-.05))^2(0.0263) + (-1-(-.05))^2(0.9737)} \approx \$5.76$$

(c) Because the sample size is large, $n = 100 > 30$, the sampling distribution of $\bar{x}$ is approximately normal with $\mu_{\bar{x}} = \mu = -\$0.05$ and $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{5.76}{\sqrt{100}} \approx \$0.576$.

(d)
$$\begin{aligned} P(\bar{x} > 0) &= P\left(Z > \frac{0-(-0.05)}{5.76/\sqrt{100}}\right) \\ &= P(Z > 0.09) \\ &= 1 - P(Z \leq 0.09) \\ &= 1 - 0.5359 \\ &= 0.4641 \quad \text{[Tech: 0.4654]} \end{aligned}$$

(e) $\mu_{\bar{x}} = -\$0.05;\ \sigma_{\bar{x}} = \frac{5.76}{\sqrt{200}} \approx \$0.407$

$$\begin{aligned} P(\bar{x} > 0) &= P\left(Z > \frac{0-(-0.05)}{5.76/\sqrt{200}}\right) \\ &= P(Z > 0.12) \\ &= 1 - P(Z \leq 0.12) \\ &= 1 - 0.5478 \\ &= 0.4522 \quad \text{[Tech: 0.4511]} \end{aligned}$$

(f) $\mu_{\bar{x}} = -\$0.05;\ \sigma_{\bar{x}} = \frac{5.76}{\sqrt{1000}} \approx \$0.275$

$$\begin{aligned} P(\bar{x} > 0) &= P\left(Z > \frac{0-(-0.05)}{5.76/\sqrt{1000}}\right) \\ &= P(Z > 0.27) \\ &= 1 - P(Z \leq 0.27) \\ &= 1 - 0.6064 \\ &= 0.3936 \quad \text{[Tech: 0.3918]} \end{aligned}$$

(g) The probability of being ahead decreases as the number of games played increases.

Section 8.2

1. 0.44; $p = \dfrac{220}{500} = 0.44$

2. sample proportion; $\dfrac{x}{n}$

3. False; while it is possible for the sample proportion to have the same value as the population proportion, it will not *always* have the same value.

4. True; the expected value, or mean, of the sampling distribution of $\hat{p}$ is the population proportion, p.

5. The sampling distribution of $\hat{p}$ is approximately normal when $n \le 0.05N$ and $np(1-p) \ge 10$.

6. The standard deviation of $\hat{p}$ decreases as the sample size increases. If the sample size is increased by a factor of 4, the standard deviation of the $\hat{p}$-distribution is reduced by a factor of 2.
$$\sigma_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$
If we increase the sample size by a factor of 4, we have $\sqrt{\dfrac{\hat{p}(1-\hat{p})}{4n}} = \sqrt{\dfrac{1}{4}} \cdot \sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}} = \dfrac{1}{2}\sigma_{\hat{p}}$
Thus, increasing the sample size by a factor of 4 will cut the standard deviation of $\hat{p}$ in half.

7. $25{,}000(0.05) = 1250$; the sample size, $n = 500$, is less than 5% of the population size and $np(1-p) = 500(0.4)(0.6) = 120 > 10$.
The distribution of $\hat{p}$ is approximately normal, with mean $\mu_{\hat{p}} = p = 0.4$ and standard deviation
$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.4(1-0.4)}{500}} \approx 0.022.$$

8. $25{,}000(0.05) = 1250$; the sample size, $n = 300$, is less than 5% of the population size and $np(1-p) = 300(0.7)(0.3) = 63 > 10$.
The distribution of $\hat{p}$ is approximately normal, with mean $\mu_{\hat{p}} = p = 0.7$ and standard deviation
$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.7(1-0.7)}{300}} \approx 0.026.$$

9. $25{,}000(0.05) = 1250$; the sample size, $n = 1000$, is less than 5% of the population size and $np(1-p) = 1000(0.103)(0.897) = 92.391 > 10$.
The distribution of $\hat{p}$ is approximately normal, with mean $\mu_{\hat{p}} = p = 0.103$ and standard deviation
$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.103(1-0.103)}{1000}} \approx 0.010.$$

10. $25{,}000(0.05) = 1250$; the sample size, $n = 1010$, is less than 5% of the population size and $np(1-p) = 1010(0.84)(0.16) = 135.744 > 10$.
The distribution of $\hat{p}$ is approximately normal, with mean $\mu_{\hat{p}} = p = 0.84$ and standard deviation
$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.84(1-0.84)}{1010}} \approx 0.012.$$

11. **(a)** $10{,}000(0.05) = 500$; the sample size, $n = 75$, is less than 5% of the population size and $np(1-p) = 75(0.8)(0.2) = 12 > 10$. The distribution of $\hat{p}$ is approximately normal, with mean $\mu_{\hat{p}} = p = 0.8$ and standard deviation
$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.8(1-0.8)}{75}} \approx 0.046.$$

(b)
$$\begin{aligned} P(\hat{p} \ge 0.84) &= P\left(Z \ge \frac{0.84-0.8}{\sqrt{0.8(0.2)/75}}\right) \\ &= P(Z \ge 0.87) \\ &= 1 - P(Z < 0.87) \\ &= 1 - 0.8078 \\ &= 0.1922 \quad \text{[Tech: 0.1932]} \end{aligned}$$
About 19 out of 100 random samples of size $n = 75$ will result in 63 or more individuals (that is, 84% or more) with the characteristic.

(c)
$$\begin{aligned} P(\hat{p} \le 0.68) &= P\left(Z \le \frac{0.68-0.8}{\sqrt{0.8(0.2)/75}}\right) \\ &= P(Z \le -2.60) \\ &= 0.0047 \end{aligned}$$
About 5 out of 1000 random samples of size $n = 75$ will result in 51 or fewer individuals (that is, 68% or less) with the characteristic.

12. (a) $25,000(0.05) = 1250$; the sample size, $n = 200$, is less than 5% of the population size and $np(1-p) = 200(0.65)(0.35) = 45.5 > 10$. The distribution of $\hat{p}$ is approximately normal, with mean $\mu_{\hat{p}} = p = 0.65$ and standard deviation

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.65(1-0.65)}{200}} \approx 0.034.$$

(b)
$$\begin{aligned} P(\hat{p} \ge 0.68) &= P\left(Z \ge \frac{0.68-0.65}{\sqrt{0.65(0.35)/200}}\right) \\ &= P(Z \ge 0.89) \\ &= 1 - P(Z < 0.89) \\ &= 1 - 0.8133 \\ &= 0.1867 \quad \text{[Tech: 0.1869]} \end{aligned}$$

About 19 out of 100 random samples of size $n = 200$ will result in 136 or more individuals (that is, 68% or more) with the characteristic.

(c)
$$\begin{aligned} P(\hat{p} \le 0.59) &= P\left(Z \le \frac{0.59-0.65}{\sqrt{0.65(0.35)/200}}\right) \\ &= P(Z \le -1.78) \\ &= 0.0375 \quad \text{[Tech: 0.0376]} \end{aligned}$$

About 4 out of 100 random samples of size $n = 200$ will result in 118 or fewer individuals (that is, 59% or less) with the characteristic.

13. (a) $1,000,000(0.05) = 50,000$; the sample size, $n = 1000$, is less than 5% of the population size and $np(1-p) = 1000(0.35)(0.65) = 227.5 > 10$. The distribution of $\hat{p}$ is approximately normal, with mean $\mu_{\hat{p}} = p = 0.35$ and standard deviation

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.35(1-0.35)}{1000}} \approx 0.015.$$

(b) $\hat{p} = \frac{x}{n} = \frac{390}{1000} = 0.39$

$$\begin{aligned} P(X \ge 390) &= P(\hat{p} \ge 0.39) \\ &= P\left(Z \ge \frac{0.39-0.35}{\sqrt{0.53(0.65)/1000}}\right) \\ &= P(Z \ge 2.65) \\ &= 1 - P(Z < 2.65) \\ &= 1 - 0.9960 \\ &= 0.0040 \end{aligned}$$

About 4 out of 1000 random samples of size $n = 1000$ will result in 390 or more individuals (that is, 39% or more) with the characteristic.

(c) $\hat{p} = \frac{x}{n} = \frac{320}{1000} = 0.32$

$$\begin{aligned} P(X \le 320) &= P(\hat{p} \le 0.32) \\ &= P\left(Z \ge \frac{0.32-0.35}{\sqrt{0.53(0.65)/1000}}\right) \\ &= P(Z \le -1.99) \\ &= 0.0233 \quad \text{[Tech: 0.0234]} \end{aligned}$$

About 2 out of 100 random samples of size $n = 1000$ will result in 320 or fewer individuals (that is, 32% or less) with the characteristic.

14. (a) $1,500,000(0.05) = 75,000$; the sample size, $n = 1460$, is less than 5% of the population size and $np(1-p) = 1460(0.42)(0.58) = 355.656 > 10$. The distribution of $\hat{p}$ is approximately normal, with mean $\mu_{\hat{p}} = p = 0.42$ and standard deviation

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.42(1-0.42)}{1460}} \approx 0.013.$$

(b) $\hat{p} = \frac{x}{n} = \frac{657}{1460} = 0.45$

$$\begin{aligned} P(X \ge 657) &= P(\hat{p} \ge 0.45) \\ &= P\left(Z \ge \frac{0.45-0.42}{\sqrt{0.42(0.58)/1460}}\right) \\ &= P(Z \ge 2.32) \\ &= 1 - P(Z < 2.32) \\ &= 1 - 0.9898 \\ &= 0.0102 \quad \text{[Tech: 0.0101]} \end{aligned}$$

About 1 out of 100 random samples of size $n = 1460$ will result in 657 or more individuals (that is, 45% or more) with the characteristic.

(c) $\hat{p} = \frac{x}{n} = \frac{584}{1460} = 0.4$

$$\begin{aligned} P(X \le 584) &= P(\hat{p} \le 0.4) \\ &= P\left(Z \ge \frac{0.4 - 0.42}{\sqrt{0.42(0.58)/1460}}\right) \\ &= P(Z \le -1.55) \\ &= 0.0606 \quad \text{[Tech: 0.0608]} \end{aligned}$$

About 6 out of 100 random samples of size $n = 1460$ will result in 584 or fewer individuals (that is, 40% or less) with the characteristic.

15. (a) The sample size, $n = 100$, is less than 5% of the population size and $np(1-p) = 100(0.82)(0.18) = 14.76 > 10$. The distribution of $\hat{p}$ is approximately normal, with mean $\mu_{\hat{p}} = p = 0.82$ and standard deviation $\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.82(1-0.82)}{100}} \approx 0.038$.

(b) $\hat{p} = \frac{x}{n} = \frac{85}{100} = 0.85$

$$\begin{aligned} P(X \ge 85) &= P(\hat{p} \ge 0.85) \\ &= P\left(Z \ge \frac{0.85 - 0.82}{\sqrt{0.82(0.18)/100}}\right) \\ &= P(Z \ge 0.78) \\ &= 1 - P(Z < 0.78) \\ &= 1 - 0.7823 \\ &= 0.2177 \quad \text{[Tech: 0.2174]} \end{aligned}$$

About 22 out of 100 random samples of size $n = 100$ Americans will result in at least 85 (that is, at least 85%) who are satisfied with their lives.

(c) $\hat{p} = \frac{x}{n} = \frac{75}{100} = 0.75$

$$\begin{aligned} P(X < 76) &= P(X \le 75) \\ &= P(\hat{p} \le 0.75) \\ &= P\left(Z \le \frac{0.75 - 0.82}{\sqrt{0.82(0.18)/100}}\right) \\ &= P(Z \le -1.82) \\ &= 0.0344 \quad \text{[Tech: 0.0342]} \end{aligned}$$

About 3 out of 100 random samples of size $n = 100$ Americans will result in fewer than 76 individuals (that is, 75% or less) who are satisfied with their lives. This result is unusual.

16. (a) The sample size, $n = 200$, is less than 5% of the population size. In addition, $np(1-p) = 200(0.8)(0.2) = 32 > 10$. The distribution of $\hat{p}$ is approximately normal, with mean $\mu_{\hat{p}} = p = 0.8$ and standard deviation $\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.8(1-0.8)}{200}} \approx 0.028$.

(b) $\hat{p} = \frac{x}{n} = \frac{154}{200} = 0.77$

$$\begin{aligned} P(X \le 154) &= P(\hat{p} \le 0.77) \\ &= P\left(Z \le \frac{0.77 - 0.8}{\sqrt{0.8(0.2)/200}}\right) \\ &= P(Z \le -1.06) \\ &= 0.1446 \quad \text{[Tech: 0.1444]} \end{aligned}$$

About 14 out of 100 random samples of size $n = 200$ college students with cell phones will result in 154 or fewer students (that is, 77% or less) who send and receive text messages. This result is not unusual.

(c) $\hat{p} = \frac{x}{n} = \frac{172}{200} = 0.86$

$$\begin{aligned} P(X \ge 172) &= P(\hat{p} \ge 0.86) \\ &= P\left(Z \ge \frac{0.86 - 0.8}{\sqrt{0.8(0.2)/200}}\right) \\ &= P(Z \ge 2.12) \\ &= 1 - P(Z < 2.12) \\ &= 1 - 0.9830 \\ &= 0.0170 \quad \text{[Tech: 0.0169]} \end{aligned}$$

About 2 out of 100 random samples of size $n = 200$ college students with cell phones will result in 172 or more students (that is, 86% or more) who send and receive text messages. This result is unusual.

17. **(a)** Our sample size, $n = 500$, is less than 5% of the population size and $np(1-p) = 500(0.26)(0.74) = 96.2 > 10$.
The distribution of $\hat{p}$ is approximately normal, with mean $\mu_{\hat{p}} = p = 0.26$ and standard deviation

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.26(1-0.26)}{500}} \approx 0.020.$$

(b)
$$\begin{aligned} P(\hat{p} < 0.24) &= P\left(Z < \frac{0.24-0.26}{\sqrt{0.26(0.74)/500}}\right) \\ &= P(Z < -1.02) \\ &= 0.1539 \quad \text{[Tech: 0.1540]} \end{aligned}$$

About 15 out of 100 random samples of size $n = 500$ adults will result in less than 24% who have no credit cards.

(c) $\hat{p} = \frac{x}{n} = \frac{150}{500} = 0.3$

$$\begin{aligned} P(X \geq 150) = P(\hat{p} \geq 0.3) &= P\left(Z \geq \frac{0.3-0.26}{\sqrt{0.26(0.74)/500}}\right) \\ &= P(Z \geq 2.04) \\ &= 1 - P(Z < 2.04) \\ &= 1 - 0.9793 \\ &= 0.0207 \end{aligned}$$

About 2 out of 100 random samples of size $n = 500$ adults will result in more than 150 who have no credit cards. This result is unusual. Thus, it is unusual for 150 of 500 adults to have no credit cards.

18. **(a)** Our sample size, $n = 750$, is less than 5% of the population size and $np(1-p) = 750(0.07)(0.93) = 48.825 > 10$.
The distribution of $\hat{p}$ is approximately normal, with mean $\mu_{\hat{p}} = p = 0.07$ and standard deviation

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.07(1-0.07)}{750}} \approx 0.009.$$

(b)
$$\begin{aligned} P(\hat{p} > 0.08) &= P\left(Z > \frac{0.08-0.07}{\sqrt{0.07(0.93)/750}}\right) \\ &= P(Z > 1.07) \\ &= 1 - P(Z \leq 1.07) \\ &= 1 - 0.8577 \\ &= 0.1423 \quad \text{[Tech: 0.1416]} \end{aligned}$$

About 14 out of 100 random samples of size $n = 750$ telephone users will result in more than 8% who are "cell-phone only."

(c) $\hat{p} = \frac{x}{n} = \frac{40}{750} = \frac{4}{75} \approx 0.053$

$$\begin{aligned} P(X \leq 40) = P(\hat{p} \leq 0.053) &= P\left(Z \leq \frac{0.053-0.07}{\sqrt{0.07(0.93)/750}}\right) \\ &= P(Z \leq -1.82) \\ &= 0.0344 \quad \text{[Tech: 0.0368]} \end{aligned}$$

About 3 out of 100 random samples of size $n = 750$ telephone users will result in 40 or fewer being "cell-phone only." This result is unusual. Thus, it would be unusual for 40 or fewer of 750 adults to be "cell-phone only."

19. $\hat{p} = \frac{x}{n} = \frac{121}{1100} = 0.11$

$$\begin{aligned} P(X \geq 121) = P(\hat{p} \geq 0.11) &= P\left(Z \geq \frac{0.11-0.10}{\sqrt{0.1(0.9)/1100}}\right) \\ &= P(Z \geq 1.11) \\ &= 1 - P(Z < 1.11) \\ &= 1 - 0.8665 \\ &= 0.1335 \quad \text{[Tech: 0.1345]} \end{aligned}$$

This result is not unusual, so this evidence is insufficient to conclude that the proportion of Americans who are afraid to fly has increased above 0.10.

20. $\hat{p} = \frac{x}{n} = \frac{328}{800} = 0.41$

$$\begin{aligned} P(X \leq 328) = P(\hat{p} \leq 0.41) &= P\left(Z \leq \frac{0.41-0.43}{\sqrt{0.43(0.57)/800}}\right) \\ &= P(Z \leq -1.14) \\ &= 0.1271 \quad \text{[Tech: 0.1266]} \end{aligned}$$

This result is not unusual, so this evidence is insufficient to conclude that the proportion of adults who have received a "phishing" contact has decreased below 0.43.

21. (a) To say the sampling distribution of $\hat{p}$ is approximately normal, we need $np(1-p) \geq 10$ and $n \leq 0.05N$.
With $p = 0.1$, we need

$$n(0.1)(1-0.1) \geq 10$$
$$n(0.1)(0.9) \geq 10$$
$$n(0.09) \geq 10$$
$$n \geq 111.11$$

Therefore, we need a sample size of 112, or 62 more adult Americans.

(b) With $p = 0.2$, we need

$$n(0.2)(1-0.2) \geq 10$$
$$n(0.2)(0.8) \geq 10$$
$$n(0.16) \geq 10$$
$$n \geq 62.5$$

Therefore, we need a sample size of 63, or 13 more, adult Americans.

22. (a) To say the sampling distribution of $\hat{p}$ is approximately normal, we need $np(1-p) \geq 10$ and $n \leq 0.05N$.
With $p = 0.9$, we need

$$n(0.9)(1-0.9) \geq 10$$
$$n(0.9)(0.1) \geq 10$$
$$n(0.09) \geq 10$$
$$n \geq 111.11$$

Therefore, we need a sample size of 112, or 12 more teenagers.

(b) With $p = 0.95$, we need

$$n(0.95)(1-0.95) \geq 10$$
$$n(0.95)(0.05) \geq 10$$
$$n(0.0475) \geq 10$$
$$n \geq 210.53$$

Therefore, we need a sample size of 211, or 111 more, teenagers.

23. A sample of size $n = 20$ households represents more than 5% of the population size $N = 100$ households in the association. Thus, the results from individuals in the sample are not independent of one another.

24. (a) – (d) Answers will vary.

(e) $\mu_{\hat{p}} = p = 0.3$;

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.3(1-0.3)}{25}} \approx 0.092$$

25. (a) – (d) Answers will vary.

26. (a) $\hat{p} = \frac{x}{n} = \frac{410}{500} = 0.82$

(b) $N = 6502$; $n = 500$; $\hat{p} = 0.82$

$$\sigma_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n-1} \cdot \left(\frac{N-n}{N}\right)}$$
$$= \sqrt{\frac{0.82(1-0.82)}{500-1} \cdot \left(\frac{6502-500}{6502}\right)}$$
$$\approx 0.017$$

Consumer Reports®: Tanning Salons

(a) Selecting a random sample from a large number of facilities increases the likelihood of obtaining an estimate of the population parameter that is close to the true value. Since the reference frame is all American tanning salons, sampling from multiple metropolitan areas ensures representation from a broad spectrum of American tanning salons.

(b) Yes, the sample size, $n = 296$, is much less than 5% of the population (150,000).

(c) The sampling distribution of $\hat{p}$ is approximately normal with a mean of $\mu_{\hat{p}} = 0.25$ and a standard deviation of

$$\sigma_{\hat{p}} = \sqrt{\frac{(0.25)(1-0.25)}{296}} \approx 0.025.$$

$$P(\hat{p} < .226) = P\left(Z < \frac{0.226 - 0.25}{\sqrt{0.25(0.75)/296}}\right)$$
$$= P(Z < -0.95)$$
$$= 0.1711 \quad \text{[Tech: 0.0.1701]}$$

About 17 out of 100 random samples of size $n = 296$ tanning facilities will result in less than 22.6% (i.e., less than 67) that would state that tanning in a salon is the same as tanning in the sun with respect to causing skin cancer.